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Chance-constrained stochastic programming under variable reliability levels with an application to humanitarian relief network design

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ABSTRACT

We focus on optimization models involving individual chance constraints, in which only the right-hand side vector is random with a finite distribution. A recently introduced class of such models treats the reliability levels / risk tolerances associated with the chance constraints as decision variables and trades off the actual cost / return against the cost of the selected reliability levels in the objective function. Leveraging recent methodological advances for modeling and solving chance-constrained linear programs with fixed reliability levels, we develop strong mixed-integer programming formulations for this new variant with variable reliability levels. In addition, we introduce an alternate cost function type associated with the risk tolerances which requires capturing the value-at-risk (VaR) associated with a variable reliability level. We accomplish this task via a new integer linear programming representation of VaR. Our computational study illustrates the effectiveness of our mathematical programming formulations. We also apply the proposed modeling approach to a new stochastic last mile relief network design problem and provide numerical results for a case study based on the real-world data from the 2011 Van earthquake in Turkey.

1. Introduction

In many decision making problems under uncertainty it is desired to account for the probability of certain unfavorable events. This is very much in agreement with the concept of reliability often used to characterize the quality of service. In a wide range of domains, applications concerned with such issues (the probability of meeting demand or the reliability of a system) give rise to mathematical programming models that involve chance (or probabilistic) constraints. Areas of application of chance-constrained optimization models include but are not restricted to production planning, supply chain management, power system planning and design, financial portfolio optimization, and humanitarian relief network design. For a thorough overview of the applications of chance-constrained optimization models and the corresponding theory and numerical methods, we refer the reader to Kall and Wallace (1994), Prékopa (1995, 2003), Dentcheva (2006), Shapiro et al. (2009), and the references therein.

There are two main types of chance constraints: joint and individual (separate) chance constraints. A joint chance constraint imposes that a set of goal constraints (inequalities) hold together with a high probability. In contrast, an individual chance constraint is introduced to account for the probability of a single goal constraint. From a modeling point of view, the problem of interest determines the appropriate type of chance constraint. As discussed in Haneveld and van der Vlerk (2015), a joint chance constraint is more fitting when the individual goal constraints collectively describe one single goal. Otherwise, if the individual goal constraints describe different goals, it makes more sense to consider them separately. In this case, the ability to vary the reliability levels associated with the separate chance constraints provides us with a flexible modeling framework, which can prioritize the set of goals at hand. In practice, another important criterion is the computational tractability of the resulting mathematical programming formulations. Optimization with a joint chance constraint is generally significantly more challenging than optimization with individual chance constraints (see, e.g., Küçükyavuz, 2012; Lejeune, 2012; Luedtke et al., 2010). Enforcing a joint chance constraint with a high probability level on a large set of goal constraints typically leads to individual probability levels close to one, and consequently, may result in very conservative solutions. As a partial

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remedy, the decision maker may opt for decreasing the probability level of the joint chance constraint, but this exacerbates the computational difficulties. Therefore, one may prefer to employ individual chance constraints even if a joint chance constraint is more appropriate from a modeling perspective. In particular, it is natural to use individual chance constraints to develop a computationally tractable approximation of a joint chance-constrained optimization model (see, e.g., Haneveld and van der Vlerk, 2015; Prékopa, 2003).

Our study is dedicated to individual chance-constrained linear programs (LP), where the uncertainty is restricted to the right-hand sides of the probabilistic constraints, and the random parameters have discrete distributions. The convexity of the feasible region is still preserved in this structure given the fixed probability levels. A general form of the classical LP with multiple individual chance constraints is then given by:

$$(\mathbf{MICC}) \quad \min_{\mathbf{x}} \quad \mathbf{c}^T\mathbf{x} \tag{1}$$

$$\text{s.t.} \quad \mathbb{P}(T_k\mathbf{x} \geq \xi_k) \geq 1-\epsilon_k, \qquad \forall\ k=1,\dots,m \tag{2}$$

$$\mathbf{x} \in \mathcal{X}. \tag{3}$$

Here, the feasible set associated with the deterministic constraints is represented by the polyhedron $\mathcal{X} \subseteq \mathbb{R}^n_+$. We assume that T is a deterministic $m \times n$ matrix with its kth row denoted by T_k, and $\boldsymbol{\xi} = (\xi_1, \dots, \xi_m)^T$ is an m-dimensional random vector with a finite support. The realizations of the random vector $\boldsymbol{\xi}$ are denoted by $\boldsymbol{\xi}^i$ with corresponding probabilities $p^i > 0$ for $i \in \{1, \dots, N\}$. The individual chance constraints (2) ensure that the stochastic (goal) constraint $T_k\mathbf{x} \geq \xi_k$ holds with a probability at least equal to $1-\epsilon_k$ for all $k \in \{1, \dots, m\}$, where ϵ_k is the risk tolerance corresponding to the reliability level $1-\epsilon_k$. Note that integrality requirements can also be incorporated into the definition of $\mathcal{X}$, and our entire modeling and solution framework directly carries over to individual chance-constrained *integer* linear programs as well.

It is well-known that (**MICC**) can be reformulated as a linear program when the risk tolerances $\epsilon_k,\ k=1,\dots,m,$ are input parameters specified by the decision maker. Alternatively, we can consider the risk tolerances / reliability levels as decision variables, and the resulting mathematical models can be put into good use in several ways at the expense of additional complexity. For instance, (**MICC**) with a set of additional constraints on the variable/adjustable reliability levels is presented in Prékopa (2003) – see formulation (5.10) – as an approximation of a joint chance-constrained optimization model. This approximation optimizes the risk tolerances and is a natural extension of the classical Bonferroni approximation, which is based on the particular choice of equal risk tolerances. Another motivation for varying the values of the reliability levels is to perform a Pareto analysis in order to extract insights about the trade-off between the actual cost/return factors and the cost associated with the probabilities of the undesirable events of interest (Rengarajan et al., 2013; Rengarajan and Morton, 2009). In a similar vein, Shen (2014) proposes a new class of optimization models with adjustable reliability levels, where the author incorporates a linear cost function of the individual risk tolerances into the objective function (1). Lejeune and Shen (2016) follow this line of research in the context of joint chance-constrained optimization. They consider two types of a joint constraint – with a deterministic technology matrix T on the left-hand side (randomness exists only on the right-hand side) and with a random technology matrix T – and develop effective mathematical programming formulations based on a Boolean modeling framework. We also refer to Lejeune and Shen (2016) for a detailed review on studies which consider a trade-off between the conflicting cost/return criteria and reliability objectives.

Our study is directly related to and motivated by Shen (2014). The author presents a mixed-integer linear programming (MIP) reformulation for (**MICC**) with adjustable risk tolerances. However, the reformulation of the chance constraints relies on the classical big-*M* paradigm and solving large instances presents a formidable challenge due to the well-known weakness of the LP relaxations of formulations featuring big-*M*s. Our primary objective in this paper is therefore to offer a computationally effective MIP reformulation of (**MICC**) when the reliability levels are treated as decision variables. To this end, we exploit recent methodological advances for modeling and solving chance-constrained linear programs with fixed reliability levels. In particular, we use a modeling approach similar to that presented in Luedtke et al. (2010). The fundamental idea is to rewrite the chance constraints (2) in the form of $T_k\mathbf{x} \geq z_k$, where z_k corresponds to the $(1-\epsilon_k)$th quantile of the random component ξ_k for $k \in \{1, \dots, m\}$. Variable reliability levels render the quantile values denoted by $z_k,\ k \in \{1, \dots, m\}$, variable as well, and reformulating the chance constraints requires being able to express the quantile values as functions of the reliability level variables. To this end, we develop two alternate approaches to express the decision variables $z_k,\ k \in \{1, \dots, m\}$. The first representation is based on the mixing inequalities proposed by Luedtke et al. (2010). The authors study the mixing set with a knapsack constraint arising in the deterministic equivalent formulation of a joint chance-constrained optimization model with a finitely distributed random right-hand side and a fixed reliability level. It turns out that the results of this work can be applied to individual chance-constrained optimization models with adjustable risk tolerances as well. An alternate second representation arises from using a different set of binary variables to identify the scenarios under which the goal constraints are violated. The resulting MIP formulations scale very well with an increasing number of scenarios and outperform the current state-of-the-art based on the big-*M* type of constraints by a significant margin – see Section 5. Therefore, one noteworthy contribution of our work is to highlight the existence and efficacy of alternate formulations for individual chance-constrained (integer) linear programs with and without variable risk tolerances and make recent methodological progress in modeling and solving chance-constrained optimization models more accessible to practitioners.

Optimization capturing the trade-off between the actual cost factors and the cost of the risk tolerances associated with the chance constraints is a fairly recent research area, and such a hybrid modeling approach has promise to be applied in different fields. In this context, we elaborate on how to construct a cost function of the variable reliability levels and extend/modify Shen (2014)'s model by quantifying the cost of reliability with a different perspective. Ultimately, we apply the proposed modeling approach to humanitarian relief logistics, where it may be essential to consider multiple and possibly conflicting performance criteria – such as accessibility and equity, see, e.g., Noyan et al. (2016). In particular, we focus on balancing the trade-off between accessibility and the level of demand satisfaction in the context of post-disaster relief network design. We introduce a new stochastic last mile distribution network design problem, which determines the locations of the Points of Distribution (PODs), the assignments of the demand nodes to PODs, and the delivery amounts to the demand nodes while considering the equity and accessibility issues and incorporating the inherent uncertainties. The studies that consider decisions related to the locations of the last mile facilities are scarce, and as emphasized in Noyan et al. (2016), the majority of these studies either assume a deterministic setting and/or do not incorporate the concepts of accessibility and equity. Our study contributes to the humanitarian relief literature by introducing a new hybrid supply allocation policy and developing a new risk-averse optimization model, which is well-solved with the proposed MIP