ARTICLE IN PRESS
JID: CAOR [m5G;November 9, 2017;19:24]
Computers and Operations Research 000 (2017) 1–20
Contents lists available at ScienceDirect
Computers and Operations Research
journal homepage: www.elsevier.com/locate/cor

Observability of power systems with optimal PMU placement

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A R T I C L E I N F O

Article history:
Received 23 February 2017
Revised 12 September 2017
Accepted 31 October 2017
Available online xxx

Keywords:
Network observability
PMU
Combinatorial optimization
Bilevel programming
Cutting plane

A B S T R A C T

Phasor Measurement Units (PMUs) are measuring devices that, when placed in electrical networks, observe their state by providing information on the currents in their branches (transmission lines) and voltages in their buses. Compared to other devices, PMUs have the capability of observing other nodes besides the ones they are placed on. Due to a set of observability rules, depending on the placement decisions, the same number of PMUs can monitor a higher or smaller percentage of a network. This leads to the optimization problem hereby addressed, the PMU Placement Problem (PPP) which aims at determining the minimum number and location of PMUs that guarantee full observability of a network at minimum cost.

In this paper we propose two general mathematical programming models for the PPP: a single-level and a bilevel integer programming model. To strengthen both formulations, we derive new valid inequalities and promote variable fixing. Furthermore, to tackle the bilevel model, we devise a cutting plane algorithm amended with particular features that improve its efficiency. The efficiency of the algorithm is validated through computational experiments. Results show that this new approach is more efficient than state-of-the-art proposals.

1. Introduction

Context. Phasor Measurement Units (PMUs) are measuring devices that, by providing time synchronized phasor measurements, allow monitoring of a electrical power network (Phadke, 1993). When installed in a bus (node) of a network, a PMU measures the voltage of that bus and the currents following across a given number of incident branches. The number of currents that can be measured depends on the number of channels of the device. Furthermore, voltages at the buses incident to those branches can be inferred from Ohm's law. For nodes with no associated load nor generation (zero-injection nodes), information about other nodes can be used by applying Kirchoff's current law. To the cascade process associated to the inference of values, we call propagation and say that a node (branch) is observed if we can infer its associated electric values.

Inference of values based on the above mentioned electrical laws allows for full monitoring (observability) of a network with a number of PMUs that is less than the number of buses. That value can be minimized if we optimize the PMU placement: depending on the placement decisions, the same number of PMUs can monitor a higher or smaller percentage of a network. The PMU Placement Problem (PPP) addresses this optimization goal: it aims at determining the minimum number of PMUs (and their location) that guarantee full observability of a network at minimum cost. The problem was proven to be NP-complete both for networks without zero-injection nodes, as it reduces to the dominating set problem (Garey and Johnson, 1979), and for the case where all nodes are zero-injection, where we have the power dominating set (Haynes et al., 2002).

Literature review. Both exact methods, based on Integer Programming (IP) and heuristics were proposed in the literature to address the PPP. In terms of IP models, Xu and Abur (2004) were the first proposing an integer nonlinear formulation to solve the problem. In their model the depth of propagation of the observability rules is limited. The model was later linearized by Dua et al. (2008) that studied the multiplicity of optimal PMU placements. Gou (2008) applied similar formulations to study the cases with redundancy and incomplete observability. Sodhi et al. (2010) also addressed the PPP. However, they only considered Ohm's law for value inference. This significantly simplifies the problem and results in solutions with a larger number of PMUs. Aminifar et al. (2010) proposed a linear IP model where

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https://doi.org/10.1016/j.cor.2017.10.012

PMUs may have a different number of channels. Later their propagation model was incorporated by Fan and Watson (2015) into a novel integer linear formulation with an exponential number of variables. PMUs with varying costs were also considered in Fan and Watson (2015). In both papers this propagation model of the observability rules is limited as proven by their computational results that attain solutions with more PMUs than others already found in the literature. Finally, Poirion et al. (2016) studied the PPP restricted to 1-channel capacity (the PMU has capacity to observe two nodes and the respective transmission line). To the best of our knowledge, this is the first study that properly models the propagation of the observability rules. They propose an integer linear program and an equivalent bilevel program. The reader is addressed to Manousakis et al. (2011); 2012) for an updated state-of-the art on formulations and solution techniques.

Besides mathematical formulations, several heuristics with an emphasis on meta-heuristics, were presented in the literature, being thoroughly reviewed in Nazari-Heris and Mohammadi-Ivatloo (2015). Some of the most relevant work mentioned in that survey is referenced next. Genetic Algorithms were used by Mohammadi-Ivatloo (2009), Marín et al. (2003) and Aminifar et al. (2009). Nuqui and Phadke (2005) combine the so called tree search placement technique with Simulated Annealing, while in Mesgarnejad and Shahrtash (2008) and Koutsoukis et al. (2013) Tabu Search approaches are presented. Hajian et al. (2007) and Ahmadi et al. (2011) designed methods based on particle swarm optimization. Finally, a Chemical Reaction method is build by Xu et al. (2013). The drawback of heuristic methods is that there is no guarantee of optimality of the solutions computed.

The computational complexity of four variants of the PPP were investigated by Gyllstrom et al. (2012): (1) minimize the number of PMUs such that full observability is guaranteed, (2) maximize the number of observed buses for a fixed number of PMUs, (3) minimize the number of PMUs such that full observability is ensured, as well as redundancy, and (4) maximize the number of observed buses for a fixed number of PMUs and ensure redundancy. Their work generalizes the complexity results by Brueni and Heath (2005), which proved that even for planar bipartite graphs the decision version of the PPP is already NP-complete.

Paper contributions and organization. In this paper we extend the work by Poirion et al. (2016) in the following directions: we adapt their IP models for the general PPP, implementing observability rules for all types of nodes and considering unconstrained PMU capacity. To strengthen the mathematical models we derive new valid inequalities and promote variable fixing. Furthermore, another type of valid inequalities fully characterizing the PPP's feasible set is derived together with a polynomial time algorithm that improves those inequalities. This leads to dominant inequalities and, as a byproduct, to an upper bound on the solution. Finally, we incorporate these ingredients in a cutting plane fashion algorithm to solve the problem at hand. We also extend those results to two problem variants: the L-capacity PMU, with $L \geq 1$, and the variable cost PMU. In this last problem we consider the case where PMUs may have different capacities and associated costs.

The paper is organized as follows. Section 2 states the PMU Placement problem, establishes general notation and presents two mathematical formulations: the observability propagation model (OPM) and a bilevel programming model. Section 3 discusses valid inequalities for the set of feasible PMU placements (*i.e.*, decision plans that lead to full observability) and variable fixing for optimal solutions. To solve the bilevel mathematical programming formulation, these theoretical results plus extra crucial enhancements are gathered to build the algorithm presented in Section 4. We validate the efficiency of our approach through computational results presented in Section 5. Section 6 summarizes our contributions and provides future research lines in this context. Flowcharts of our algorithms can be found in Appendix A.

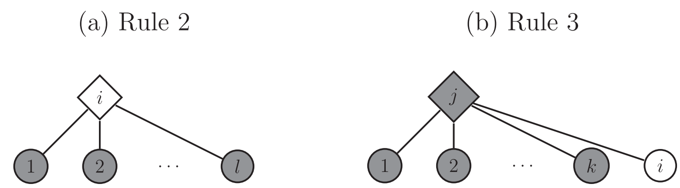

Fig. 1. Diamonds represent zero-injection nodes and circles are any node in V (zero- or non zero-injection). The gray nodes are observed; the white nodes become observed by (a) Rule 2, (b) Rule 3.

2. Problem statement and formulations

Consider a power system network (PSN) that can be represented by an undirected graph $G = (V, E)$, where V and E respectively represent sets of buses (nodes) and transmission lines (edges). We assume that if $(i, j) \in E$ then $(j, i) \in E$. The neighborhood $N(i)$ for $i \in V$ is the set of buses adjacent to i: $N(i) = \{j | (i, j) \in E\}$, and define $N[i] = N(i) \cup \{i\}$. Furthermore, the set of nodes V in G can be formally partitioned into two subsets: the set of nodes with no associated generation or load, so called zero-injection nodes, and the set of nodes with associated generation or load, non zero-injection nodes.

A phasor measurement unit (PMU) is a device that allows to monitor the state of a PSN. It can measure the voltage of the bus it is placed in and the current in all the adjacent lines. A bus is said to be observed if its voltage is know, while a transmission line is observed if its current is known. A PSN is fully observed if the voltage and currents in all its buses and transmission lines are known. Furthermore, based on two fundamental electrical circuit laws, Ohm's law and Kirchoff's current law, additional voltages and currents can be obtained.

- **Ohm's law** states that the current I_{ij} in line (i, j) is $I_{ij} = \frac{V_i - V_j}{R_{ij}}$, where R_{ij} is the resistance of the line, and V_i, V_j are the potentials (voltages) of nodes i and j, respectively.
- **Kirchhoff's current law** states that for a zero-injection bus i, $\sum_{j \in N(i)} I_{ij} = 0$.

These laws support the following set of observability rules where, for simplicity of explanation, we assume that PMUs have unlimited capacity and can observe an unlimited number of elements of the network.

Rule 1: A node i is observed if node i or one of its neighbors has a PMU;
Rule 2: A zero-injection node i is observed if all its neighbors are observed (see Fig. 1 (a));
Rule 3: If a zero-injection node j and all its neighbors, except for node i, are observed then i is observed (see Fig. 1(b)).

These rules have a cascade propagation nature. Fig. 2 illustrates observability propagation, for a given PSN with PMUs placed in nodes 1 and 9. Initially (step 0), due to rule 1 nodes 2, 3, 7 and 8 are observed. Then, in a second step, as 1,2, and 3 are observed, by rule 3 node 4 becomes observed. Further, Rule 2 is applied to node 6 as 4, 7 and 8 are observed. Finally, in the last step, again by rule 3, node 5 becomes observed.

Definition 2.1. The PMU placement problem (PPP) can be defined as follows: find a placement for the minimum number of PMUs that ensures full observability.

In the remaining of this section we propose generalizations of the two formulations presented in Poirion et al. (2016) and discuss

Please cite this article as: M. Carvalho et al., Observability of power systems with optimal PMU placement, Computers and Operations Research (2017), https://doi.org/10.1016/j.cor.2017.10.012