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p-hub median problem for non-complete networks

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ABSTRACT

Most hub location studies in the literature use a complete-network structure as an input in developing optimization models. This starting point is not necessarily from assuming that the underlying real-world network (e.g., physical network such as road and rail networks) on which the hub system will operate is complete. It is implicitly or explicitly assumed that a complete-network structure is constructed from the shortest-path lengths between origin-destination pairs on the underlying real-world network through a shortest-path algorithm. Thus, the network structure used as an input in most models is a complete network with the distances satisfying the triangle inequality. Even though this approach has gained acceptance, not using the real-world network and its associated data structure directly in the models may result in several computational and modeling disadvantages. More importantly, there are cases in which the shortest path is not preferred or the triangle inequality is not satisfied. In this regard, we take a new direction and define the p-hub median problem directly on non-complete networks that are representative of many real-world networks. The proposed problem setting and the modeling approach allow several basic assumptions about hub location problems to be relaxed and provides flexibility in modeling several characteristics of real-life hub networks. The proposed models do not require any specific cost and network structure and allow to use the real-world network and its asociated data structure directly. The models can be used with the complete networks as well. We also develop a heuristic based on the proposed modeling aproach and present computational studies.

1. Introduction

Hub facilities exist in many telecommunication and transportation systems where commodities (data, packages, passengers, etc.) are sent between many origin-destination (OD) pairs. In such systems, instead of establishing direct links (i.e., fully interconnected network) and sending flows between all OD pairs directly, some or all commodities are sent through one or more hubs that act as sorting, switching, connecting, and consolidation points.

A generic hub location problem involves determining the locations of hub facilities and the assignment of service routes between OD pairs. Different types of hub location problems, e.g., p-hub median, p-hub center, and hub covering, have been defined and extensively studied in the literature. See, for instance, Campbell et al. (2001) and Alumur and Kara (2008) for a review. However, several researchers, e.g., Campbell and O'Kelly (2012) and Contreras (2015), emphasize that there is a need to go beyond the classical hub location problems and define new ones that better represent the real-world hub systems. Specifically, Campbell and O'Kelly (2012) state that much more effort should be directed at better modeling of specific air, ground, or water transportation systems and operations and incorporating both more realistic transportation costs and service measures along with other relevant aspects into models rather than solving very large-scale idealized hub location problems.

The source of motivation for our study is this stated need. We aim to develop a problem setting and modeling approach that will allow some limiting assumptions underlying most hub location models to be relaxed and hence to add flexibility and realism in modeling several characteristics of real-life hub systems. In the paper, we focus on the p-hub median version of the problem but our approach can be adapted to other hub location problems as well.

For a detailed discussion about the assumptions of the hub location models, see, e.g., Campbell and O'Kelly (2012), Contreras (2015), and Campbell et al. (2015). In the following, we discuss some assumptions (properties) together with their direct and indirect consequences and then explain how we address them.

To start with, we define five types of networks: (1) *Real-world network* (RealN): The physical network, e.g., road and rail networks,

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on which the hub system will operate. (2) *Modeled network* (MN): The network used as an input in developing a model for the problem. MN is not necessarily the same as RealN but may be obtained from RealN through preprocessing. (3) *Hub network* (HN): The subnetwork of MN that consists of the hub nodes, non-hub nodes, and the arcs on the service routes between OD pairs. (4) *Hub-level network* (HLN): The subnetwork of HN consisting of the hub nodes and the arcs connecting them. (5) *Access network* (AN): The subnetwork of HN consisting of the hub nodes, non-hub nodes, and *access arcs* that connect non-hub origin and destination nodes to hub nodes.

Most models assume that *MN is a complete network with arc distances satisfying the triangle inequality* ***(Assumption 1)***. This assumption is not necessarily a result of RealN because RealN may not be complete as is the case in most real-life networks, e.g., rail and road networks (or even if RealN is complete, its distances may not satisfy the triangle inequality). In such cases, even though not generally stated in most studies, researchers make an implicit assumption that a complete MN is constructed from the *shortest path lengths between OD pairs in RealN* through an algorithm such as the Floyd Algorithm (Floyd, 1962) (and hence arc distances satisfy the triangle inequality). However, this approach and the assumption have some implications and disadvantages: (1) The model size gets large very quickly. In an n-node complete network, the number of arcs is $n(n-1)/2$ while it is at most $3n-6$ in a planar network (Nishizeki and Chiba, 1998). (2) It is difficult to handle some situations, e.g., when some arcs in RealN are appropriate for the passage of small vehicles but not for large vehicles that are used for inter-hub transport. (3) The routing information on RealN can only be obtained after post-processing the solution on HN because *an arc in a complete MN* (and hence in HN) *may actually correspond to a shortest path and not necessarily a single arc in RealN.* (4) It is more challenging to model arc capacities on RealN and the interactions between facility location and routing decisions. (5) The approach cannot be applied for cases in which (i) using the shortest path costs is not necessarily preferred or correct, e.g., communication networks, and (ii) expecting the triangle inequality to hold is not realistic, e.g., passenger airline, train, and bus fares. (6) Cost model cannot take into account the fact that cost factors and their effects on different arcs of RealN may be different. In general, a standard transportation rate per unit distance per unit flow in all arcs is used. However, the incurred costs change depending on several factors. For example, costs are different for large- and small-size vehicles on different types of roads with different speed limits, congestion levels, and tolls, e.g., Transportation Research Board (2013) and Ricardo-AEA (2014). ((7) Cases in which there are multiple arcs between two nodes in RealN with different costs and service levels cannot be modeled. As an example, there are three options for the drivers in Turkey to move from one side of the Gulf of Izmit to the other, which is on the route between Istanbul and the cities in the west of Turkey: (i) using toll ferry with a travel time (cost) of about 1 h ($25), (ii) driving around the gulf and through city center with a travel time (cost) of about 2 h ($10), and (iii) using the toll bridge with a travel time (cost) about 4–6 min. ($30). (8) Some definitions (e.g., *service level* or *network topology*) may become vague for some cases. For example, if there are hop constraints (e.g., the number of arcs on a route) for a road network, it is not the actual number of arcs in RealN but the shortest-path arcs in MN considered with the current approach if additional data and constraints are not used.

The aforementioned issues clearly indicate that there are several advantages of eliminating Assumption 1 and using RealN and its associated data structure directly in MN, which is what we do in this paper. Specifically, we directly define the p-hub median problem on *non-complete networks* that are representative of most RealNs. This allows us to develop *models that do not require any specific cost and network structures*. Currently, there are no such models; all models use a complete network structure as MN. Marin et al. (2006) modify some hub location models to make them usable when the triangle inequality is not satisfied. However, modified models are based on other limiting assumptions to be mentioned.

There are three movement types in a hub system: *collection* from the origin to a hub, *transfer* between hubs, and *distribution* from the last hub to the destination. Most studies assign *constant cost rates* (per unit distance per unit flow) χ, α, and δ for collection, transfer, and distribution on all *arcs*, respectively, with $\alpha < \chi$ and $\alpha < \delta$ to capture *economies of scale*. In general, $\chi = \delta = 1$ and $0 \le \alpha \le 1$. Thus, *transportation costs on all inter-hub arcs are discounted by a constant factor of* α *independent of the actual amount of flow on the arcs* ***(Assumption 2)***. Most models also assume that *all flows are routed via a set of hubs (no direct flows between non-hub nodes)* (***Assumption 3***). These three assumptions without any other restrictions imply that transportation cost is minimized when *flows between OD pairs visit at most two hubs in a HN* (***Property 1***), i.e., a route between an OD pair in an HN consists of at most three arcs, namely, *collection* (*access*), *transfer* (*hub*), and *distribution* (*access*) arcs. Given non-zero flows between all OD pairs, Property 1 in turn implies that *all hubs are fully interconnected by the hub arcs*, i.e., *HLN is a complete network* (***Property 2***).

Most studies including Marin et al. (2006) impose these two properties as topological requirements on HN. However, there may be cases where these properties are restrictive. For example, when the distances in MN do not satify the triangle inequality, there may be more than two hubs in the optimal routes. Similarly, if the setup costs for the hub arcs are significant, the complete-HLN topology may not be appropriate. Several studies (e.g., Campbell et al., 2005; Alumur et al., 2009; Calik et al., 2009; Yaman, 2009; Contreras et al., 2010; Campbell, 2010; Martins de Sá et al., 2013; Martins de Sá et al., 2015a, 2015b) have tried to address these shortcomings by allowing different HLN or AN topologies. However, they are restrictive as well due to Assumption 1, e.g., a tree-like HLN may not be constructed correctly when the triangle inequality is not satisfied.

Another issue related to Assumption 2 and Property 2 is about the modeling of economies of scale. The costs on all hub arcs are discounted by a factor of α because it is assumed that the flows are concentrated on the hub arcs. However, an analysis of the optimal flows indicates that some access arcs carry much higher flows than some hub arcs, but the costs on the access arcs are not discounted in the assumed cost structure. To remedy this weakness, some researchers (e.g., O'Kelly and Bryan [O'Kelly and Bryan, 1998]) develop models with *flow-dependent discounts on the hub arc costs*. Campbell et al. (2005a, 2005b) *locate the hub arcs* with each end point being a hub that may or may not be connected and thus *relax Property 2*. Campbell (2013) analyzes the optimal flows on the hub and access arcs by using different models with different data sets. He finds out that all models allow the access arc flows to exceed the hub arc flows at varying degrees and concludes that the requirement of Property 2 and the lack of discounts on the access arcs create the poor modeling of economies of scale rather than the form of cost discounting (flow-independent or flow-dependent) used for hub arc flows.

With these results in mind, we do not impose any HLN and AN topologies in our modeling approach. Costs on all hub arcs need not necessarily be discounted and costs on the access arcs may be discounted if necessary; it is possible to assign different discount rates for each type of movement (collection, transfer, and distribution) on each arc in RealN. We build our basic model with *flow-independent costs* on the hub and access arcs but show how to extend it to handle *flow-dependent costs*. The proposed approach is