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Packing unequal rectangles and squares in a fixed size circular container using formulation space search

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A B S T R A C T

In this paper we formulate the problem of packing unequal rectangles/squares into a fixed size circular container as a mixed-integer nonlinear program. Here we pack rectangles so as to maximise some objective (e.g. maximise the number of rectangles packed or maximise the total area of the rectangles packed). We show how we can eliminate a nonlinear maximisation term that arises in one of the constraints in our formulation. We indicate the amendments that can be made to the formulation for the special case where we are maximising the number of squares packed. A formulation space search heuristic is presented and computational results given for publicly available test problems involving up to 30 rectangles/squares. Our heuristic deals with the case where the rectangles are of fixed orientation (so cannot be rotated) and with the case where the rectangles can be rotated through ninety degrees.

1. Introduction

In this paper we consider the problem of packing non-identical rectangles (i.e. rectangles of different sizes) into a fixed size circular container. Since the circular container may not be large enough to accommodate all of the rectangles available to be packed there exists an element of choice in the problem. In other words we have to decide which of the rectangles will be packed, and moreover for those that are packed their positions within the container. The packing should respect the obvious constraints, namely that the packed rectangles do not overlap with each other and that each packed rectangle is entirely within the container. This packing should be such so as to maximise an appropriate objective (e.g. maximise the number of rectangles packed or maximise the total area of the rectangles packed).

To illustrate the problem suppose we have ten rectangles with sizes as shown in Table 1 to be packed into a fixed sized circular container. The rectangles shown in Table 1 have been ordered into ascending area order.

Regarding the rectangles as being of fixed orientation, i.e. they cannot be rotated, then:

- If we are wish to maximise the number of rectangles packed Fig. 1 shows the solution as derived by the approach presented in this paper. In that figure we can see that seven of the ten rectangles have been packed, three rectangles are left unpacked.
- If we are wish to maximise the total area of the rectangles packed Fig. 2 shows the solution as derived by the approach presented in this paper. In that figure we can see that five of the ten rectangles have been packed.

If the rectangles can be rotated through ninety degrees then:

- If we are wish to maximise the number of rectangles packed Fig. 3 shows the solution as derived by the approach presented in this paper. In that figure we can see that seven of the ten rectangles have been packed, three rectangles are left unpacked.
- If we are wish to maximise the total area of the rectangles packed Fig. 4 shows the solution as derived by the approach presented in this paper. In that figure we can see that seven of the ten rectangles have been packed.

In Figs. 3 and 4 the letter r after the rectangle number indicates that the rectangle has been rotated through ninety degrees. Comparing Figs. 1 and 3 we can see that they both involve the packing of seven rectangles. Whilst allowing rotation through ninety degrees allows the possibility of a better solution as compared with the no rotation case this is by no means assured. Comparing Figs. 2 and 4 we can see that in this particular case an improvement in the total area of the rectangles packed has been made by making use of rotation.

The structure of this paper is as follows. In Section 2 we review the literature relating to the packing of rectangles. We discuss ap-

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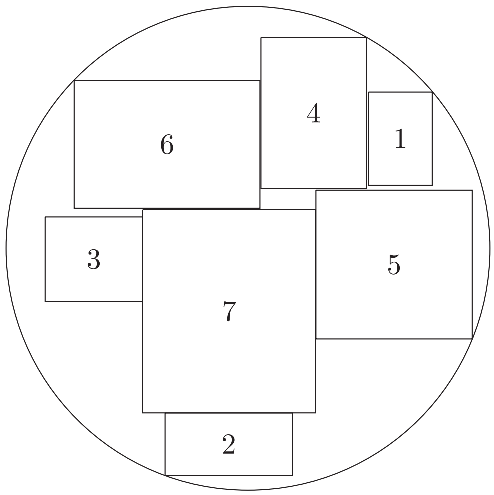

Fig. 1. Maximise the number of rectangles packed, no rotation, solution value 7.

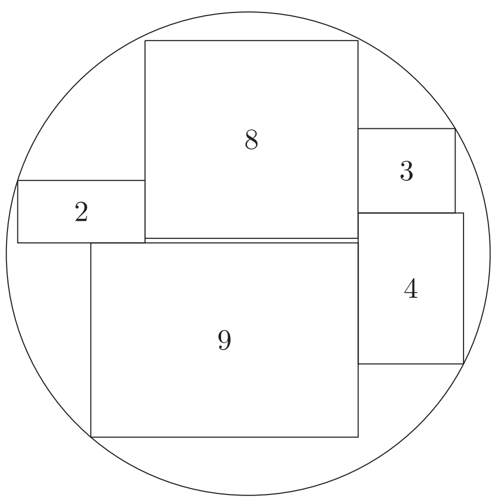

Fig. 2. Maximise the total area of the rectangles packed, no rotation, solution value 37.6878.

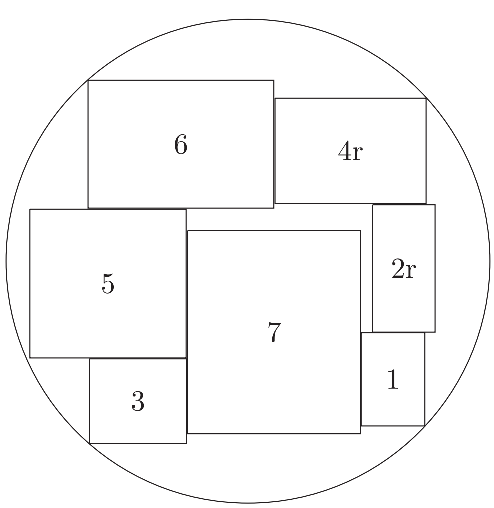

Fig. 3. Maximise the number of rectangles packed, rotation allowed, solution value 7.

Table 1
Rectangle packing example, circular container radius 4.18.

Rectangle	Length	Width
1	1.10	1.61
2	2.20	1.08
3	1.68	1.46
4	1.82	2.61
5	2.70	2.57
6	3.21	2.21
7	2.99	3.51
8	3.68	3.42
9	4.62	3.36
10	3.79	4.79

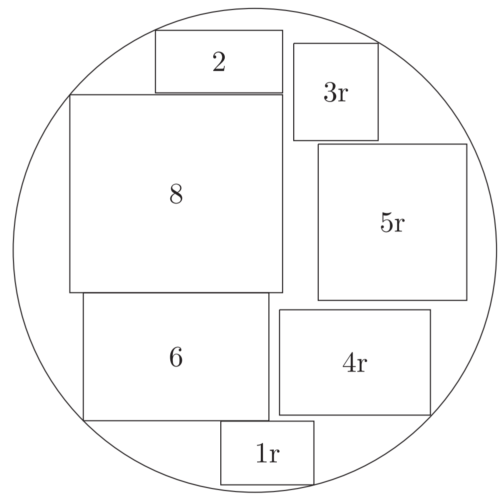

Fig. 4. Maximise the total area of the rectangles packed, rotation allowed, solution value 37.9687.

plication areas where rectangle packing problems arise. We also review the literature relating to the particular metaheuristic, formulation space search, used in this paper. In Section 3 we formulate the problem of packing unequal rectangles/squares into a fixed size circular container as a mixed-integer nonlinear program. We show how we can eliminate a nonlinear maximisation term that arises in one of the constraints in our formulation. We also show how we can deal with the case where rectangles can be rotated through ninety degrees. We indicate the amendments that can be made to the formulation for the special case where we are maximising the number of squares packed. Section 4 gives details of the formulation space search heuristic that we use to solve the problem. Computational results are presented in Section 5 for problems involving up to 30 rectangles/squares. In that section we give results both for maximising the number of rectangles/squares packed and for maximising the total area of the rectangles/squares packed. Finally in Section 6 we present our conclusions.

2. Literature survey

In this section we first discuss the literature relating to the problem of packing rectangles and its applications. We then discuss the literature relating to the particular metaheuristic, formulation space search, we use to solve the rectangle packing problem considered in this paper.

2.1. Rectangle packing

The majority of the work in the literature related to rectangle packing deals with packing rectangles/squares within a larger