Computers and Operations Research 94 (2018) 118–138

Contents lists available at ScienceDirect

Computers and Operations Research

journal homepage: www.elsevier.com/locate/cor

A cellular memetic algorithm for the examination timetabling problem

Nuno Leite[a,b,*], Carlos M. Fernandes[b], Fernando Melício[a,b], Agostinho C. Rosa[c,b]

[a] ISEL – Instituto Superior de Engenharia de Lisboa, Instituto Politécnico de Lisboa, Rua Conselheiro Emídio Navarro, n.° 1, Lisboa 1959-007, Portugal
[b] LARSyS: Laboratory for Robotics and Systems in Engineering and Science, Universidade de Lisboa, Av. Rovisco Pais, n.° 1, Lisboa 1049-001, Portugal
[c] Department of Bioengineering/Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, n.° 1, Lisboa 1049-001, Portugal

ARTICLE INFO

Article history:
Received 28 May 2015
Revised 16 February 2018
Accepted 17 February 2018
Available online 20 February 2018

2010 MSC:
90-08
90B40
90C27
90C35
90C59

Keywords:
Cellular evolutionary algorithm
Examination timetabling
ITC 2007 benchmark set
Memetic algorithms
Threshold acceptance algorithm
Timetabling
Uncapacitated Toronto benchmark set

ABSTRACT

The timetabling problem involves the scheduling of a set of entities (e.g., lectures, exams, vehicles, or people) to a given set of resources in a limited number of time slots, while satisfying a set of constraints. In this paper, a cellular memetic algorithm is proposed for solving the examination timetabling problem. Cellular evolutionary algorithms are population-based metaheuristics. They differ from non-cellular algorithms in that the population is organised in a cellular structure, providing for a smooth actualisation of the populations that contributes to improving the population diversity. The proposed cellular evolutionary algorithm is hybridised with the threshold acceptance local search metaheuristic. The implemented algorithm uses feasible genetic recombination and local search operators, thus limiting the exploration to the feasible solution space. The effect of the threshold acceptance used in the hybrid algorithm for the examination timetabling problem is studied. It is shown that a low threshold decreasing rate is needed in order to rearrange the most difficult exams in better periods, allowing for the easy set of exams to be placed in good periods as well. The approach was tested on the public Toronto and ITC 2007 benchmark sets. The proposed hybrid is able to attain four and three new upper bounds for the Toronto and ITC 2007 benchmark sets, respectively.

1. Introduction

Transportation companies, educational, health, and sport institutions, all have to solve timetabling problems several times in a year. Due to the combinatorial nature of timetabling problems, solving them for relatively large size institutions is a complex task. For this reason, efficient tools for constructing feasible and optimal (or near-optimal) timetables automatically are asked for by decision makers.

In timetabling, one has to schedule a set of events (lectures, exams, surgeries, sport events, trips) using a set of resources (teachers, nurses and medical doctors, referees, vehicles) over space (classrooms, examination rooms, operating rooms, sport fields), in a given period of time. For instance, in the *examination timetabling problem* (ETP) (Qu et al., 2009), the goal is to allocate exams and corresponding enrolled students to examination rooms over time periods. Additionally, a set of *hard* and *soft* constraints are considered. The hard constraints must be satisfied in order to have a feasible timetable; on the other hand, there is no obligation to satisfy the soft constraints, and violations of these may occur. The optimisation goal is usually the minimisation of the soft constraints violations.

Examples of hard constraints include: schedule all exams (the timetable must be *complete*), do not exceed room capacity, guarantee room exclusiveness for given exams, guarantee that no students will attend two or more exams in the same time slot, guarantee exam ordering (e.g., exam *A* should be placed after exam *B*), etc. (McCollum et al., 2012). Real examination timetabling problems include the following soft constraints: avoid students being enrolled in two exams the same day and in consecutive periods, avoid students having examinations in distinct periods within a given gap (period spread constraint), allocate exams with more students enrolled at the beginning of the timetable to allow for exam grading and proofing, among others. The ETP is further classified as Uncapacitated ETP (if the room capacity is unlimited) or Capacitated ETP (if the room capacity is limited) (Qu et al., 2009).

* Corresponding author at: ISEL – Instituto Superior de Engenharia de Lisboa, Instituto Politécnico de Lisboa, Rua Conselheiro Emídio Navarro, n.° 1, Lisboa 1959-007, Portugal.

E-mail addresses: nleite@cc.isel.ipl.pt (N. Leite), cfernandes@laseeb.org (C.M. Fernandes), fmelicio@laseeb.org (F. Melício), acrosa@laseeb.org (A.C. Rosa).

https://doi.org/10.1016/j.cor.2018.02.009
0305-0548/

The ETP can be modelled as a multi-objective problem since several objectives are considered (reflecting the interests of the various stakeholders such as students, institution decision makers, and teachers) (Burke et al., 2008). However, due to its computational complexity, the ETP has been addressed as a single-objective problem (McCollum et al., 2012; Qu et al., 2009) or as a two-objective problem (Cheong et al., 2009; Côté et al., 2004), where the second objective is the minimisation of the timetable length. In standard benchmarks, such as the Toronto and ITC 2007 sets, there are essentially two goals, achieving feasibility, i.e. obtain a hard constraint cost of zero, and minimising the soft constraint cost.

The ETP belongs to the NP-complete class of problems (de Werra, 1985, 1997). It has been approached mainly by mathematical programming methods (for relatively small size instances), and by approximate methods such as Artificial Intelligence (Schaerf, 1999) methods and metaheuristics (Qu et al., 2009).

Examination timetable optimisation is mainly conducted offline. In fact, in educational institutions, timetables are constructed weeks before the beginning of the school period. This is done to allow students to plan the course work during the period (semester, quarter, etc.). Due to the ETP computational complexity, execution times of algorithms could last days in order to produce good quality timetables for a large instance. Hence, there are usually two main indicators for assessing the overall performance: (1) solution quality, and (2) algorithm time performance. These indicators are inversely proportional to each other, in the sense that if one wants to achieve better results, a longer execution time is needed in order to explore a larger set of solutions and vice versa.

1.1. Our contribution

In this paper, a novel *cellular evolutionary algorithm* (cEA) (Alba and Dorronsoro, 2005) is proposed for solving the examination timetabling problem. The proposed hybrid solution approach includes an improvement step, where the *threshold acceptance* (TA) (Dueck and Scheuer, 1990) metaheuristic is applied after the variation operators (mutation and crossover operators). We show in this study that by combining the global and local search algorithms of *memetic algorithms* (MA) (Neri et al., 2012), we achieved an algorithmic design which is superior to local search and evolutionary algorithm alone. Moreover, it is shown that the local search step is of vital importance in order to schedule the most difficult exams in good positions, allowing for the later scheduling of easier exams. Using the proposed hybrid approach, an effective balance between exploration and exploitation is achieved. To the best of our knowledge, cellular evolutionary algorithms with TA have not previously been applied successfully to the ETP.

The proposed method is a two-phase algorithm (Lewis, 2008). In the first phase a set of feasible solutions is constructed, whereas in the second phase the solutions are optimised by the proposed metaheuristic.

The key contributions of this work are as follows: 1) the introduction of an evolutionary algorithm, based on the cellular model, for solving examination timetabling problems. The cellular model maintains a greater population diversity as compared to the standard evolutionary algorithm, which allows to find better quality solutions (Alba and Dorronsoro, 2005). This model is suited to be implemented in parallel, although this point was not explored, and the algorithm was implemented in a sequential way; (2) a study of the local search impact on the scheduling of the difficult and easy exams' sets (classified based on the exams' hard constraints) for the instances of the Toronto benchmark set. For this set, we consider the most difficult exams to be the ones that have more conflicts with other events; and (3) a study on the performance of the solution method, including a comparison with state-of-the-art algorithms on standard public benchmarks, namely the Toronto data set and the *Second International Timetabling Competition* (ITC 2007) data set. On the Toronto data set, the proposed approach obtains results that improve four of the previous best known results. On the ITC 2007 data set, the solution approach yields three new best solutions.

The remainder of the paper is organised as follows. The related work is surveyed in Section 2. Section 3 describes the uncapacitated Toronto and capacitated ITC 2007 problem instances. Section 4 presents the cellular evolutionary algorithm developed for the examination timetabling problem. Section 5 reports the experimental results. Some conclusions are drawn in Section 6.

2. Related work

This section provides an overview of some techniques that are related to our work. Welsh and Powell (1967) establish a relation between graph colouring and timetabling. In examination timetabling problems, the exams are represented by vertices in a graph, and an edge connects any two vertices having common students. This basic framework only enforces one hard constraint (the *clash* constraint), which guarantees that no student will attend two or more exams in the same period. The additional soft and hard constraints are considered separately and evaluated to obtain the solution fitness (Qu et al., 2009). The graph colouring problem consists of assigning colours to vertices, so that no adjacent vertices have the same colour. In our context, it corresponds to assigning timeslots to exams, while guaranteeing that the hard clash constraint is satisfied. Several graph colouring heuristics proposed (e.g., the saturation degree heuristic (Brélaz, 1979)) were applied to the ETP (e.g., as in Carter (1986)).

Thompson and Dowsland (1998) present a *simulated annealing* (SA) (Dowsland and Thompson, 2012) approach to the ETP. The authors compare three neighbourhood operators (*standard* – where the neighbourhood comprises the set of solutions produced by modifying the colour of a single vertex, *Kempe chains*, and *S-Chains*) and conclude that the operator based on Kempe chains is the most effective. The algorithm was tested on eight ETP instances from different universities. Mühlenthaler (2015) investigates the structure of the course timetabling search space and establishes sufficient conditions for the connectedness of clash-free timetables under the Kempe-exchange operation. The SA metaheuristic was applied to the high school timetabling problem by Melício et al. (2000). The authors claim that any SA algorithm depend on how the structural elements are defined (i.e., solution space, generation of new solutions, cost function) and present a comparison for several parameters of the algorithm. In a later work (Melício et al., 2004), the same authors analyse two well-known neighbourhood operators adapted to the school timetabling problem. The authors demonstrate that, for the studied problem, the double move intra-class neighbourhood always showed a better performance than the single move neighbourhood, even if the latter is improved with a heuristic method. The tests were made using real data from three different Portuguese schools. Burke and Bykov (2008) employ a variant of *hill-climbing* (HC) to include a so-called *late-acceptance* (LA) strategy, which accepts a neighbouring solution evaluated several iterations before. Burke et al. (2010) investigate the use of hybrid variable neighbourhood approaches to university exam timetabling.

Colorni et al. (1991) investigates the application of *genetic algorithms* (GA) to the timetable case. The application of a GA to timetabling and scheduling is also studied by Fang (1994). Wong et al. (2002) apply a GA to solve the ETP. They present an exam timetable automation tool based on a genetic algorithm. A