Computers and Operations Research 91 (2018) 112–120

Contents lists available at ScienceDirect

Computers and Operations Research

journal homepage: www.elsevier.com/locate/cor

An efficient exact model for the cell formation problem with a variable number of production cells

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ARTICLE INFO

Article history:
Received 7 April 2016
Revised 27 August 2017
Accepted 13 November 2017
Available online 16 November 2017

Keywords:
Cell formation problem
Cellular manufacturing
Fractional objective
Two-index model
Grouping efficacy

ABSTRACT

The Cell Formation Problem has been studied as an optimization problem in manufacturing for more than 90 years. It consists of grouping machines and parts into manufacturing cells in order to maximize loading of cells and minimize movement of parts from one cell to another. Many heuristic algorithms have been proposed which are doing well even for large-sized instances. However, only a few authors have aimed to develop exact methods and most of these methods have some major restrictions such as a fixed number of production cells for example. In this paper we suggest a new mixed-integer linear programming model for solving the cell formation problem with a variable number of manufacturing cells. The popular grouping efficacy measure is used as an objective function. To deal with its fractional nature we apply the Dinkelbach approach. Our computational experiments are performed on two testsets: the first consists of 35 well-known instances from the literature and the second contains 32 instances less popular. We solve these instances using CPLEX software. Optimal solutions have been found for 63 of the 67 considered problem instances and several new solutions unknown before have been obtained. The computational times are greatly decreased comparing to the state-of-art approaches.

1. Introduction

The Cell Formation Problem as a part of Group Technology (GT) was introduced by Burbidge (1961) and Mitrofanov (1966). In the most general formulation it is designed to reduce production costs by grouping machines and parts into manufacturing cells (production shops). The goal of such kind of grouping is to set up manufacturing process in a way that maximizes loading of machines within the cells and minimizes movement of parts from one cell to another. In classical formulation the problem is defined by a binary matrix A with m rows representing machines and p columns representing parts. In this machine-part matrix $a_{ij} = 1$ if part j is processed on machine i. The objective is to form production cells, which consist of machines and parts together, optimizing some production metrics such as machine loading and intercell movement.

As an example of input data we will consider the instance of Waghodekar and Sahu (1984) shown in Table 1. This instance consists of 5 machines and 7 parts. The ones in a machine-part matrix are called *operations*. In Table 2 a solution with 2 manufacturing cells is presented. The first manufacturing cell contains machines m_1, m_4 with parts p_1, p_7 and the second manufacturing cell contains machines m_2, m_3, m_5 with parts p_2, p_3, p_4, p_5, p_6. Some parts have to be moved from one cell to another for processing (e.g. part p_6 needs to be processed on machine m_1, so it should be transported from its cell 2 to cell 1). The operations lying outside cells are called *exceptional elements* or *exceptions*. There can be also non-operation elements inside cells ($a_{ij} = 0$). These elements reduce machine load and are called *voids*. So the goal is to minimize the number of exceptions and the number of voids at the same time.

1.1. Related work

Many different approaches are proposed for solving the cell formation problem. The majority of them provide heuristic solutions and only a few exact methods have been suggested.

Krushinsky and Goldengorin (1984) provided two MINpCUT exact models based on the well-known k-cut graph partition problem. The objective function considered in this research is minimization of the exceptional elements number for a fixed number of cells. Unfortunately this objective function does not address the load inside cells.

Elbenani and Ferland (2012) presented a mixed-integer linear programming model which maximizes the most popular objective for the cell formation problem - the grouping efficacy, introduced

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https://doi.org/10.1016/j.cor.2017.11.009
0305-0548/

Table 1
Machine-part 5 × 7 matrix from Waghodekar and Sahu (1984).

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
m_1	1	0	0	0	1	1	1
m_2	0	1	1	1	1	0	0
m_3	0	0	1	1	1	1	0
m_4	1	1	1	1	0	0	0
m_5	0	1	0	1	1	1	0

Table 2
Solution with 2 production cells.

	p_7	p_1	p_6	p_5	p_4	p_3	p_2
m_1	1	1	1	1	0	0	0
m_4	0	1	0	0	1	1	1
m_2	0	0	0	1	1	1	1
m_3	0	0	1	1	1	1	0
m_5	0	0	1	1	1	0	1

by Kumar and Chandrasekharan (1990). These authors suggested to apply Dinkelbach algorithm since the grouping efficacy is a fractional objective function. Their model allows solving the cell formation problem only if the number of production cells is predefined. Thus the suggested approach cannot guarantee global optimality of the obtained solutions with respect to a variable number of production cells. In many cases the computational times for this model are quite long or memory limitations are exceeded and the optimal solutions cannot be found.

Brusco (2015) introduced two approaches for solving the cell formation problem with the grouping efficacy objective. The first is a mixed-integer linear programming model which is based on a general two-mode clustering formulation with some simplifying assumptions (e.g. the numbers of clusters by rows and columns are equal). This model looks interesting, but requires too much time to be solved for many even medium-sized instances. The second approach is a branch-and-bound algorithm combined with a relocation heuristic to obtain an initial solution. The branch and bound approach is able to solve about two times more problem instances and the computational times are greatly improved as well. Generally it runs fine on well-structured (with grouping efficacy value $>$ 0.65–0.7) medium-sized problems. Two major assumptions are made for both of these approaches: singletons are permitted (manufacturing cells containing only one machine or one part) that is quite a common practice; residual cells are permitted (cells containing only machines without parts, or only parts without machines). Also the number of production cells is predefined for the both approaches, but for some test instances several values for the number of cells are considered in computational experiments.

Another model is provided in our earlier paper (Bychkov et al., 2014). There we present a mixed-integer linear programming formulation for the cell formation problem with a variable number of production cells. It deals well with small-sized instances, but nevertheless the number of variables and constraints is huge - $O(m^2p)$. This does not allow obtaining solutions even for some moderate-sized test instances and in some cases this model runs too slowly.

Some authors used biclustering approaches to solve the cell formation problem. Boutsinas (2013) applied simultaneous clustering for both dimensions (machines and parts) and minimized the number of voids plus the number of exceptional elements. Pinheiro et al. (2016) reduced the cell formation problem to another biclustering problem - bicluster graph editing problem and suggested an exact method and a linear programming model which provides good computational results for the grouping efficacy objective.

Table 3
Testset A - instances.

ID	Source	m	p
A1	King and Nakornchai (1982) - Fig. 1a	5	7
A2	Waghodekar and Sahu (1984) - Problem 2	5	7
A3	Seifoddini (1989b)	5	18
A4	Kusiak and Cho (1992)	6	8
A5	Kusiak and Chow (1987)	7	11
A6	Boctor (1991) - Example 1	7	11
A7	Seifoddini and Wolfe (1986)	8	12
A8	Chandrasekaran and Rajagopalan (1986a)	8	20
A9	Chandrasekaran and Rajagopalan (1986b)	8	20
A10	Mosier and Taube (1985a)	10	10
A11	Chan and Milner (1982)	15	10
A12	Askin and Subramanian (1987)	14	24
A13	Stanfel (1985)	14	24
A14	McCormick et al. (1972)	16	24
A15	Srinivasan et al. (1990)	16	30
A16	King (1980)	16	43
A17	Carrie (1973)	18	24
A18	Mosier and Taube (1985b)	20	20
A19	Kumar et al. (1986)	23	20
A20	Carrie (1973)	20	35
A21	Boe and Cheng (1991)	20	35
A22	Chandrasekharan and Rajagopalan (1989) - Dataset 1	24	40
A23	Chandrasekharan and Rajagopalan (1989) - Dataset 2	24	40
A24	Chandrasekharan and Rajagopalan (1989) - Dataset 3	24	40
A25	Chandrasekharan and Rajagopalan (1989) - Dataset 5	24	40
A26	Chandrasekharan and Rajagopalan (1989) - Dataset 6	24	40
A27	Chandrasekharan and Rajagopalan (1989) - Dataset 7	24	40
A28	McCormick et al. (1972)	27	27
A29	Carrie (1973)	28	46
A30	Kumar and Vannelli (1987)	30	41
A31	Stanfel (1985) - Fig. 5	30	50
A32	Stanfel (1985) - Fig. 6	30	50
A33	King and Nakornchai (1982)	30	90
A34	McCormick et al. (1972)	37	53
A35	Chandrasekharan and Rajagopalan (1987)	40	100

1.2. Contributions of this research

In this paper we develop a fast compact model providing optimal solutions for the cell formation problem with a variable number of manufacturing cells and the grouping efficacy objective. Unlike the majority of linear programming models our model does not contain a direct assignment of machines or parts to cells. We use machine-machine and part-machine assignments instead of the widely used machine-part-cell assignment. This leads to a compact and elegant formulation considering only constraints which ensure a block-diagonal structure of solutions. It allows us to drastically reduce the number of variables and constraints in our programming model and obtain globally optimal solutions even for some large-sized problem instances.

Computational experiments show that our model outperforms all known exact methods. We have solved 63 of 67 problem instances to the global optimum with respect to a variable number of production cells. We have also found several new solutions unknown before.

We would like to highlight that many researchers in the field use the 35 GT instances dataset provided by Gonçalves and Resende (2004). These instances are taken from different cell formation research papers (references to the original sources are shown in Table 3). Some problem instances in this 35 GT dataset have errors and differ from the ones presented in the original papers. Many researchers including Elbenani and Ferland (2012) and Pinheiro et al. (2016) have performed their computational experiments using these data from Gonçalves and Resende (2004). We have reviewed all the original sources, comparing and forming the corrected version of this popular dataset. We have also collected many other problem instances less popular and formed a new