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Increasing discrimination of DEA evaluation by utilizing distances to anti-efficient frontiers

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ABSTRACT

This paper develops three DEA performance indicators for the purpose of performance ranking by using the distances to both the efficient frontier and the anti-efficient frontier to enhance discrimination power of DEA analysis. The standard DEA models and the Inverted DEA models are used to identify the efficient and anti-efficient frontiers respectively. Important issues like possible intersections of the two frontiers are discussed. Empirical studies show that these indicators indeed have much more discrimination power than that of standard DEA models, and produce consistent ranks. Furthermore, three types of simulation experiments under general conditions are carried out in order to test the performance and characterization of the indicators. The simulation results show that the averages of both the Pearson and Spearman correlation coefficients between true efficiency and indicators are higher than those of true efficiency and efficiency scores estimated by the BCC model when sample size is small.

1. Introduction

Data envelopment analysis (DEA) was first introduced by Charnes et al. [10], and has been widely used in performance or productivity evaluation. The main idea of the classic DEA is to first identify the production frontier on which the decision making units (DMUs) will be regarded as efficient. Then those DMUs not on the frontier will be compared with their peers on the frontier to estimate their efficiency scores. All the DMUs on the frontier are deemed to have the same level of performance and to represent the best practice. One of the main advantages of DEA is to allow the DMUs to have full freedom to select their weights, which are most favorable for their assessments to achieve the maximum efficiency score. This full flexibility of selecting weights is important in the identification of inefficient DMUs. However, this full flexibility may much reduce the discrimination power of DEA in the sense that there often exist too many DMUs on the frontier, which cannot be further ranked in the standard DEA models. When there are many input and output variables but only a few DMUs are available, decision makers (DMs) may find that all or most DMUs are efficient, and such results would be of little use for decision making. As [1], p. 250 argued, "*Often decision-makers are interested in a complete ranking, beyond the dichotomized classification, in order to refine the evaluation of the units.*"

Regarding the number of DMUs required in DEA models, [11], p. 252 proposed a rule of thumb, which demands

$$n \geq \max\{m \times s, 3(m+s)\},$$

where n is the number of DMUs, m and s are the number of inputs and outputs. However, the rule above is sometimes violated in reality, because of small DMUs sample but many input and output variables. In such case, the standard DEA models are not as useful as expected.

Therefore, many researchers have sought to improve the discrimination capability of standard DEA models. Now there are three main areas in DEA literature: The first area requires preferential or prior information from relevant decision-makers to enhance the discrimination ability of DEA models. For example, some scholars have developed the weights restriction [2,27] or preference change methods [17,21,32] to incorporate the prior information or value judgments of DMs into DEA models. The second area is based on cross-efficiency matrix, in which DMUs are evaluated by both itself and other peers [13,23]. Although cross-efficiency method is often very useful, in our opinion, the cross-efficiency scores have moved quite away from the basic principle of DEA. For instance in the case of one standard input variable, all the DMUs in fact use the same weights to compute their cross-efficiency scores. The third popular area is the super-efficiency method, which computes the score of

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the DMU being evaluated by excluding itself from the reference set [4,5]. It is clear that this model uses different reference sets to evaluate the efficient DMUs and inefficient DMUs. Furthermore, Banker and Chang [7] reported that Andersen and Petersen's [4] procedure using the super-efficiency scores for ranking efficient observations had poor performance.

Whilst each technique is useful in a specialist area, no one can be referred to as a complete solution to all problems. In this paper we explore another idea to enhance the discrimination power of DEA. People often have more than one reference point of view in judging DMUs. That is they do not just compare the DMUs with good references, but sometimes with bad references as well. In other words, on one hand a DMU is better if it is closer to the good references (or efficient frontier); on the other hand, it is also good if it is far from the bad references (or anti-efficient frontier). In this sense, the standard DEA models have just employed the best practice DMUs to construct the efficient frontier and have not fully taken the advantage of the information implied in the data. The earliest work on anti-efficient frontier can be traced to "Inverted" DEA model proposed by Yamada et al. [30]. Compared to the standard DEA models which evaluate DMUs from the perspective of optimism, "Inverted" DEA model is to evaluate the performance of DMUs from the perspective of pessimism. Recently, some scholars employed Inverted DEA model to exploit more information from the data in their applications. For example, Takamura and Tone [25] employed the DEA and Inverted DEA with weights restriction to solve the problem of site selection for the relocation of several government agencies outside of Tokyo. Paradi et al. [22] used DEA and Inverted DEA models, which are so called "Worst Practice DEA" in the paper, to identify the worst practices in banking credit analysis. By using some layering or peeling technique [26], the proposed approach increased the classification accuracies through the elimination of self-identifiers. Johnson and McGinnis [16] employed both the efficient and anti-efficient frontiers to identify outliers and thus improve the accuracy of estimators in the second stage regression analysis.

In addition, some scholars tried to construct some new efficiency measures based on DEA and Inverted DEA models. Entani et al. [14] employed both DEA and Inverted DEA models to obtain the upper and lower bound of interval efficiency of DMUs. They argued if the range of the interval efficiency is large, then it means that although the DMU performs good from the optimistic viewpoint, it performs bad from the perspective of pessimistic. Then they used the interval efficiency to obtain a partial-order relation of DMUs. [28,29] constructed the best and worst virtual DMUs as TOPSIS does [15], and simply add them into the existing DMU set to carry out further DEA and Inverted DEA analysis using the extended data set. However, it may not be a wise idea because the Production Possibility Set (PPS) will be greatly changed in this case. Amirteimoori [3],[1] employed the Inverted DEA models to define the anti-efficient frontier. Then he used slacks based DEA and Inverted models to measure the weighted L_1–distances from DMU_0 to both efficient and anti-efficient frontier. Finally, he defined a new combined efficiency measure based on the two distances to rank DMUs. However, since the efficiency scores of these DMUs on efficient frontier and anti-efficient frontier are 1 and −1 respectively, this combined efficiency measure is not able to improve discrimination power of DEA models either. Furthermore, there is no justification that the combined efficiency measure performs better than existing ones. Cao et al. [9] uses the evidential-reasoning (ER) approach to construct a performance indicator for combining the efficiency and anti-efficiency obtained by DEA and inverted DEA models. Zhou et al. [33] used the DEA model without explicit inputs (see, e.g., [19,20,31]) to combine the efficient and anti-efficient measures to rank the DMUs. However we can easily verify that their approach cannot increase significantly the discrimination power of DEA models.

In this paper, we develop another DEA approach based on the idea of utilizing both good and bad frontiers to enhance discrimination power of DEA. The remainder of the paper is organized as follows: In Section 2, we discuss the approaches that can identify the anti-efficient frontier of DMUs. Furthermore, in this section, we introduce three composite performance indicators to combine the information from both best and worst viewpoints, as well as the dealing of DMUs on both efficient and anti-efficient frontiers; In Section 3, we provide two empirical studies to illustrate the features of the indicators, and then we carry out simulation studies to examine the performance of our composite indicators in Section 4. Finally, conclusions and discussions are given in Section 5.

2. Ranking DMUs via both efficient and anti-efficient frontiers

In this section we first outline our approach. For simplicity, we will illustrate the idea based on the radial measurement. Let $X = (x_1, x_2, \dots, x_m)$ and $Y = (y_1, y_2, \dots, y_s)$ be input and output vectors of m and s dimension respectively. Then Production Possibility Set (PPS) is defined by

$$PPS = \{(X, Y):\ X \text{ can produce } Y\}.$$

The boundary of PPS is referred to as production technology or production frontier. Note, this unobservable production frontier is called true frontier or true efficient frontier hereinafter. When output is single, the production frontier is called production function in economic literature. DMUs which are technically efficient operate along the frontier, while those technically inefficient DMUs operate at points in the interior of PPS. Thus it is rational to rank DMUs according to their distances to the true frontier.

Let $\{(x_j, y_j)|j = 1, \dots, n\}$ be a group of observed input and output data. Based on such observations, DEA models construct a piecewise linear production frontier, a non-parametric estimate of the unobservable true frontier. Then DEA models measure the efficiency of a DMU via its distance to the estimated frontier. Here we restate the input-oriented CCR model with slacks of inputs and outputs as follows.

$$h_b^* = \min \quad \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$
$$s.t. \begin{cases} \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta x_{i0}, \ i = 1, \dots, m, \\ \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{r0}, \ \ r = 1, \dots, s, \\ \lambda_j, s_i^-, s_r^+ \geq 0, j = 1, \dots, n. \end{cases} \tag{1}$$

where ε is a non-Archimedean infinitesimal.

In theory, Banker [6] provided a formal statistical foundation for DEA and argued that while the efficient frontier is biased below the true efficient frontier for a finite sample size, the bias goes zero for large samples. However when sample size is small, the estimated frontier could be far away from the true one so that the efficiency scores of DMUs are much higher than their true efficiency scores. For instance, many DMUs are on the estimated frontier and cannot be discriminated although some of them are in fact quite far from the true frontier.

[1] Note: In our view, there are some typos (errors) on inequalities in model (8) and model (10) in Amirteimoori [3].