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Modeling and optimizing traffic light settings in road networks

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ABSTRACT

We discuss continuous traffic flow network models including traffic lights. A mathematical model for traffic light settings within a macroscopic continuous traffic flow network is presented, and theoretical properties are investigated. The switching of the traffic light states is modeled as a discrete decision and is subject to optimization. A numerical approach for the optimization of switching points as a function of time based upon the macroscopic traffic flow model is proposed. The numerical discussion relies on an equivalent reformulation of the original problem as well as a mixed-integer discretization of the flow dynamics. The large-scale optimization problem is solved using derived heuristics within the optimization process. Numerical experiments are presented for a single intersection as well as for a road network.

1. Introduction

In the last few decades, time-dependent models for transportation have been of interest to researchers in different disciplines and application areas. Currently, time-dependent transportation models are used to describe, for example, data flow in telecommunication networks [21], gas or water flow in connected pipe systems [4,5,16], evacuation problems [34], and production processes (see [2,20,29,36,43]). We are interested in time-dependent models for traffic flow in road networks using continuous models. These models belong to the class of macroscopic models, and the governing equations are inspired by the equations of gas dynamics, *not*considering individual drivers but describing traffic by a continuum. Starting with the work of Lighthill, Whitham and Richards [47,55], such conservation laws describing traffic phenomena have been discussed intensively in recent years, and we refer to [7,13,15,37,38,40,45] for a non-exhaustive list treating the case of traffic road networks in particular. The macroscopic (or continuous) models typically describe the temporal and spatial evolution of traffic density and do not predict the traffic patterns of individuals. Most of the existing continuous models consider unidirectional traffic, and therefore, the traffic density depends only on a single spatial dimension in such models. Research in past years has concentrated on road networks, with an emphasis on boundary conditions for the density. These conditions are typically imposed as couplings for the vehicle flow across intersections [15] and will be discussed below in detail (see, also, [3,11,14,33,48,60]).

Concerning the study of traffic lights (and the control thereof), there exists a broad literature (see, for example, [1,6,8,10,12,22,23,28,33,35,39,44,46,48–50,53,54,61,59]). The discussion depends on the model used to predict traffic flow. The approaches range from cellular automaton to macroscopic models, depending on the spatial and temporal resolution of the model. Typically, the studied models attempt to either minimize the travel time (of individual drivers) or maximize the total traffic flow throughput at a given intersection. The majority of approaches use the cycle length (the time difference between the red and green phase) as a control parameter.

Two widely used adaptive traffic control systems are the “split cycle offset optimization technique” (SCOOT) [41] and “Sydney coordinated adaptive traffic systems” (SCATS) [51]. These systems use traffic detectors to adapt the length of the green and red phases according to the actual traffic situation. The SCOOT is based on the offline signal optimization tool TRANSYT [57], which uses [39] on microscopic traffic models and is one of the most widely used signal optimization packages.

As in [25], we restrict our discussion to complex urban intersections. To the best of our knowledge, little research has been performed on optimal traffic light settings for pde-based macroscopic network models. Regardless of the description of the traffic density, macroscopic models can capture qualitative phenomena, such as the propagation of traffic jams. Here, we combine this model based on partial differential equations with discrete decisions on switching times. The combination of optimization techniques for pde-based problems and integer restrictions on variables has been studied in the field of production in [24,26,29,30,63]. We proceed by transforming the model into a linear mixed-integer programming problem. In this

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way, it is possible to add integer restrictions and, furthermore, to study Branch-and-Bound strategies to find optimal solutions. However, the high number of variables and constraints resulting from the time and space discretization is too large to simply apply a Branch-and-Bound-based black box solver. For this reason, we investigate strategies that make use of the knowledge of the model description to speed up the optimization.

The presented approach is a model-based offline optimal control approach that may be used for planning purposes or for providing a priori settings within a suitable domain of interests. We assume knowledge about incoming flows and traffic distribution rates at branching points for the full control period.

The outline of this work is as follows: In Sections 2 and 3, we recall the basic traffic flow network model and introduce the modeling of traffic lights at intersections. Then, in Section 4, we equivalently rewrite the extended traffic model to obtain a linear mixed-integer problem (MIP) and provide techniques for the optimization procedure in Section 5. Finally, in Section 6.2, we present the numerical results obtained using different parameter settings and highlight the effect of additional constraints to avoid undesired oscillations at switching points. We compare the performance of the solution process, incorporating several techniques to reduce the computation time.

2. Modeling road networks with traffic lights

We model a road network as a directed graph $G=(V,E)$, where E denotes the set of edges representing roads and V denotes the set of vertices representing traffic intersections (also called junctions). The length of each unidirectional road i is denoted by $L_i \in \mathbb{R}^+$. These sets are referred to as $E^{in} \subset E$ or $E^{out} \subset E$. For a fixed vertex v, the set of incoming and outgoing edges are denoted by δ_v^{in} and δ_v^{out}, respectively. In the following, $t \geq 0$ denotes time, and $x \in [0, L_i]$ denotes the position within road i.

The traffic flow on each road is described by a macroscopic traffic flow model given by a hyperbolic partial differential equation (PDE) for the traffic density $\rho(x,t)$ [7,13,15,27,45]. We assume that the average car velocity is only a function of the density (number of cars per unit length). The arising (first-order) hyperbolic equation was first proposed by Lighthill, Whitham and Richards and is given in general form by Eq. (2). A junction is located at either $x=0$ or $x=L_i$, leading to boundary values for (2). These are obtained implicitly in terms of coupling conditions [27,40]. Due to the macroscopic model, all traffic lights affect only the traffic density and flux. We choose the maximization of the traffic flow $f(\rho_i(x,t))$ on each connected road i to model the least possible delay for drivers.

$$\max \sum_{i \in E}\left(\int_0^T \int_0^{L_i} f(\rho_i(x,t))\,dx\,dt + \int_0^T \hat{\gamma}_i(t)\,dt\right), \tag{1}$$

where $\rho(x,t) \in [0, \rho^{max}]$ denotes the density of cars at position x, and $\hat{\gamma}_i$ characterizes the flow of cars through junctions and will be defined below. The maximization problem is solved subject to constraints on the evolution of the density along roads (i), the coupling conditions at junctions (ii) and the traffic light modeling (iii).

(i) *Car density evolution along the roads*: The dynamics are governed by a first-order hyperbolic traffic flow model:

$$\begin{cases} \partial_t \rho_i + \partial_x f_i(\rho) = 0, \\ \rho_i(x,0) = \rho_i^0(x). \end{cases} \tag{2}$$

The flow function $f(\rho)$ is assumed to be concave, with a unique maximum at $\rho^* \in [0, \rho^{max}]$ (see [47]). As in [14,18,19,52], we use a piecewise constant symmetric triangular flow function (3):

$$f(\rho) = \begin{cases} \lambda \cdot \rho & \text{if } 0 \leq \rho \leq \rho^* \\ \lambda \cdot (2\rho^* - \rho) & \text{if } \rho^* < \rho \leq \rho^{max}, \end{cases} \tag{3}$$

with constant velocity λ, maximal density ρ^{max} and $\rho^* = \frac{1}{2}\rho^{max}$. Let the function $\rho \to \tau(\rho)$ for each $\rho \neq \rho^*$ be given by $f(\rho) = f(\tau(\rho))$, with $\tau(\rho) \neq \rho$. In the case of (3), $\tau(\rho)$ is obviously given by

$$\tau(\rho) = 2\rho^* - \rho. \tag{4}$$

(ii) *Coupling conditions at junctions and boundary values for* (2): For a fixed vertex v, we denote the density of the incoming road i by $\hat{\rho}_i(t) := \rho(x=L_i, t)$ and the density of the outgoing road j by $\overline{\rho}_j(t) := \rho(x=0,t)$. Then, we assume that Kirchoff's condition (5) holds true.

$$\sum_{i \in \delta_v^{in}} f(\hat{\rho}_i(t)) = \sum_{j \in \delta_v^{out}} f(\overline{\rho}_j(t)), \quad \forall t > 0, \quad \forall v \in V. \tag{5}$$

We define the incoming and outgoing flows of the junction as $\hat{\gamma}_i(t) := f(\hat{\rho}_i(t))$ and $\overline{\gamma}_j(t) := f(\overline{\rho}_j(t))$, respectively. It is well known [27] that (5) is not sufficient to obtain the boundary conditions $\hat{\rho}_i$ and $\overline{\rho}_j$ for (2). Additional conditions need to be imposed to obtain a well-posed problem. Similar to [7,27,37], we prescribe additional flow distribution parameters $0 \leq d_{ij} \leq 1$ at each junction v. Here, d_{ij} models the percentage of drivers traveling from edge i to edge j, and therefore, we assume that $\sum_{j \in \delta_v^{out}} d_{ij} = 1$ and that

$$\overline{\gamma}_j(t) = \sum_{i \in \delta_v^{in}} d_{ij} \cdot \hat{\gamma}_i(t), \quad \forall j \in \delta_v^{out}. \tag{6}$$

Furthermore, an entropy condition (12a) is imposed to maximize the local flux at the junction. We need to obtain the boundary values for the density $(\hat{\rho}_i, \overline{\rho}_j)$ for the connected roads $i \in \delta_v^{in}$ and $j \in \delta_v^{out}$ from the local fluxes $(\hat{\gamma}_i, \overline{\gamma}_j)$ at the junction. To obtain well-posed boundary values, the following intervals (7) and (8) for the densities are admissible:

$$\hat{\rho}_i \in \begin{cases} \{\rho_i(L_i)\} \cup]\tau(\rho_i(L_i)), \rho_i^{max}], & \text{if } 0 \leq \rho_i(L_i) \leq \rho_i^* \\ [\rho_i^*, \rho_i^{max}], & \text{else} \end{cases} \tag{7}$$

for incoming roads $i \in \delta_v^{in}$ and

$$\overline{\rho}_j \in \begin{cases} [0, \rho_j^*], & \text{if } 0 \leq \rho_j(0) \leq \rho_j^*, \\ \{\rho_j(0)\} \cup [0, \tau(\rho_j(0))[, & \text{else} \end{cases} \tag{8}$$

for outgoing roads $j \in \delta_v^{out}$.

The orientation of the square brackets indicates whether the interval is open or closed. Clearly, the previous constraints impose constraints on the fluxes $(\hat{\gamma}_i, \overline{\gamma}_j)$ at the intersection. The admissible fluxes are therefore determined by

$$\hat{\gamma}_i \leq \hat{F}_i := \begin{cases} f_i(\rho_i(L_i)) & \text{if } 0 \leq \rho_i(L_i) \leq \rho_i^* \\ f_i(\rho_i^*) & \text{else.} \end{cases} \tag{9}$$

and

$$\overline{\gamma}_j \leq \overline{F}_j := \begin{cases} f_j(\rho_j^*) & \text{if } 0 \leq \rho_j(0) \leq \rho_j^* \\ f_j(\rho(0)) & \text{else.} \end{cases} \tag{10}$$

A general existence result in the case of a single junction under the conditions (2), (5), (6), (9), (10) and (12a) using a wave-front tracking algorithm was presented in [15].

(iii) *Modeling of the influence of traffic lights*: A traffic light is located at the end of a road i. Its influence on the macroscopic traffic density is modeled by piecewise constant functions $A_i : t \mapsto \mathbb{B}$, where $\mathbb{B} := \{0,1\}$. We set $A_i(t) = 0$ if the traffic light is red at time t and $A_i(t) = 1$ if it is green. The light controls the throughput of the possible incoming flow, and therefore, we also impose the following for all