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On a doubly dynamically controlled supermarket model with impatient customers

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ABSTRACT

In this paper, we provide a key generalization of the supermarket model both from the impatient customers and from a doubly dynamic control, which may also be related to the size-based scheduling through the centered management of the customer resource as well as the total service ability. We first use an infinite-dimensional Markov process to express the states of this supermarket model, and set up an infinite-dimensional system of differential equations satisfied by the expected fraction vector. Then we use the operator semigroup to provide a mean-field limit for the sequence of infinite-dimensional Markov processes, which asymptotically approaches a single trajectory identified by the unique and global solution to the infinite-dimensional system of limiting differential equations. Finally, we provide an effective and efficient algorithm for computing the fixed point of the infinite-dimensional system of limiting differential equations, and use the fixed point to give performance analysis of this supermarket model. Also, some numerical examples are given to demonstrate how the performance measures depend on some crucial parameters of this supermarket model. We believe that the mean-field method developed in this paper will be useful and effective for analyzing more complicated supermarket models in many practical areas.

1. Introduction

Dynamic randomized load balancing is often referred to as the supermarket model. The supermarket models have a wide range of practically applicable areas such as call centers, health care, computer networks, production/inventory systems, and transportation networks. Recently, the supermarket models have been analyzed by means of queueing methods as well as Markov processes. For a simple supermarket model, Vvedenskaya et al. [50] applied the operator semigroup and mean-field limit to compute the stationary queue length distribution of any queueing process, and obtained its doubly exponential decay tail which is a substantial improvement of system performance over that in the ordinary M/M/1 queue. At nearly the same time, Mitzenmacher [37] also analyzed the same supermarket model in terms of the density-dependent jump Markov processes. Turner [48] provided a martingale approach to further discuss the supermarket model. The path space evolution of the supermarket model was studied by Graham [23,24] who showed that starting from independent initial states, as $N\to\infty$ the queues of the limiting process evolve independently. Luczak and McDiarmid [33] showed that the length of the longest queue scales as $(\log\log N)/\log d+O(1)$. Certain generalization of the supermarket model has been explored in studying various variations, for example, modeling more crucial factors by Vvedenskaya and Suhov [51], Mitzenmacher [38], Mitzenmacher et al. [39], Bramson et al. [10–12], Li and Lui [30,31], Li et al. [32,29] and Li [27,28]; fast Jackson networks by Martin and Suhov [35], Martin [34] and Suhov and Vvedenskaya [46].

Queues with impatient customers represent a wide range of service systems in which customers may become impatient when they do not receive service fast enough, and they are always useful in modeling many practical situations such as banks, hospitals, supermarkets, supply chains and transportation systems. Readers may refer to some important publications for more details, among which are the M/M/1 queue with impatient customers by Choi et al. [16], Brandt and Brandt [14] and Altman and Yechiali [1]; the M/M/c queue with impatient customers by Stanford [44], Bhattacharya and Ephremides [5], Boxma and de Waal [7], Movaghar [40], Brandt and Brandt [13], Boots and Tijms [6], Yechiali [52] and Perel and Yechiali [43]; the GI/M/s queue with impatient customers by Teghem [47], Swensen [45] and Perry et al. [42]; the M/G/1 queue with impatient customers by de Kok and Tijms [21], Bae et al. [4], Perry and Asmussen [41], Martin and Artalejo [36], Boxma et al. [8], Iravani and Balcıoğlu [25] and Boxma et al. [9];

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and the GI/G/1 queue with impatient customers by Daley [19], Baccelli and Hébuterne [3] and Baccelli et al. [2]. The matrix-analytic method is applied to study queues with impatient customers, e.g., see Combé [18], Choi et al. [17], Van Velthoven et al. [49] and Chakravarthy [15]. In applications of queues with impatient customers, examples include Kaspi and Perry [26] for inventory systems, Zohar et al. [54] for call centers, and Zeltyn and Mandelbaum [53] for telecommunication networks.

The purpose of this paper is to provide a key generalization of the supermarket model both from the impatient customers and from a doubly dynamic control. The generalized supermarket model is described as a queueing network which consists of one server and N waiting lines with impatient customers under two centered management modes: Each arriving customer joins the shortest one among the d_1 randomly selected waiting lines (JSQ (d_1)), and the server provides its service in the longest one among the d_2 randomly selected waiting lines (SLQ (d_2)). Firstly, such two centered management modes may be related to the size-based scheduling both for a suitable allocation of customer resource and for a total use of multiple service units. Then our numerical examples indicate that for improving system performance, the SLQ(d_2) controlling the service processes is more effective than the JSQ(d_1) controlling the arrival processes. Finally, it is necessary to analyze the useful relations among some special cases. When $d_2=1$ and the impatience rate $\theta=0$, this supermarket model is equivalently degraded to an ordinary supermarket model, that is, each waiting line has a server with exponential service times of rate μ. When $d_1=1$ and $d_2=1$, this supermarket model is equivalent to a system of N independent M/M/1 queues with impatient customers.

The main contributions of this paper are twofold. The first one is to provide a key generalization of the supermarket model both from the impatient customers and from a doubly dynamic control. We use the operator semigroup to provide the mean-field limit for the sequence of infinite-dimensional Markov processes, and give a computational framework for organizing a Lipschitzian condition, which is always a key for proving existence and uniqueness of solution to the infinite-dimensional system of differential equations by means of the Picard approximation. The second contribution of this paper is to provide an effective and efficient algorithm for computing the fixed point, which leads to performance analysis of this supermarket model. Also, some numerical examples are used to indicate how the performance measures depend on some crucial parameters of this supermarket model. It is worthwhile to note that the results given in this paper are lucky from analysis of a key generalized supermarket model, and also improve some numerical research from the double choice numbers $d_1 \geq 1$ and $d_2 \geq 1$, since only one choice of the parameter, i.e., $d=2$ was always assumed in the literature, e.g., see Vvedenskaya and Suhov [51] and Mitzenmacher et al. [39]. Note that the double choice numbers $d_1 \geq 1$ and $d_2 \geq 1$ are respectively related to the size-based scheduling through the centered management of the customer resource as well as the total service ability, thus this paper provides some new highlight on analyzing more complicated supermarket models from the size-based scheduling, e.g., see Dell'Amico et al. [20].

The remainder of this paper is organized as follows. In Section 2, we describe a supermarket model with impatient customers under a doubly dynamic randomized load balancing control, and use an infinite-dimensional Markov process to express the state of the supermarket model. In Section 3, we set up an infinite-dimensional system of differential equations, which is satisfied by the expected fraction vector. In Section 4, we apply the operator semigroup to provide a mean-field limit for the sequence of infinite-dimensional Markov processes, which asymptotically approaches a single trajectory identified by the unique and global solution to the infinite-dimensional system of limiting differential equations. In Section 5, we provide a computational framework for organizing a Lipschitzian condition, which is always a key for proving existence and uniqueness of solution to the infinite-dimensional system of differential equations by means of the Picard approximation. In Section 6, we provide an effective and efficient algorithm for computing the fixed point. In Section 7, we use the fixed point give performance analysis of this supermarket model, and provide some numerical examples to analyze how the performance measures depend on some crucial parameters of this supermarket model. Some concluding remarks are given in the final section.

2. A supermarket model with impatient customers

In this section, we describe a supermarket model with impatient customers, which consists of one server and N waiting lines under a doubly dynamic randomized load balancing control. We use the fraction vector of waiting lines with at least k customers for $k \geq 0$ to organize an infinite-dimensional Markov process who expresses the state of the supermarket model at time $t \geq 0$.

2.1. Model description

Let us describe the supermarket model with impatient customers, which consists of one server and N waiting lines under a doubly dynamic randomized load balancing control, where the random factors of this supermarket model are listed as follows:

The arrival process and its dynamically control: Customers arrive at the supermarket model as a Poisson process with arrival rate $N\lambda$ for $\lambda > 0$. Upon arrival, each customer chooses $d_1 \geq 1$ waiting lines from the N waiting lines independently and uniformly at random, and joins the one whose queue length is the shortest among the d_1 selected waiting lines. If there is a tie, waiting lines with the shortest queue are chosen randomly.

The service process and its dynamical control: There is one server working among the N waiting lines in this system, and the service times are i.i.d. and is exponential with service rate N for $\mu > 0$. The server always chooses $d_2 \geq 1$ waiting lines independently and uniformly at random from the N waiting lines, and provides its service in the one whose queue length is the longest among the d_2 selected waiting lines.

The impatient process: The waiting customers excluding the one in service are impatient, and each of them has an exponential sojourn time with impatience rate $N\theta$ for $\theta > 0$. Obviously, if there is a customer at the server and $k-1$ customers in the waiting room (note that there are k customers in the system), then $(k-1)N\theta$ is the total impatience rate due to the independently impatient behavior of the $k-1$ customers.

We assume that all the random variables defined above are independent of each other. Fig. 1 provides a physical illustration of the supermarket model with impatient customers.

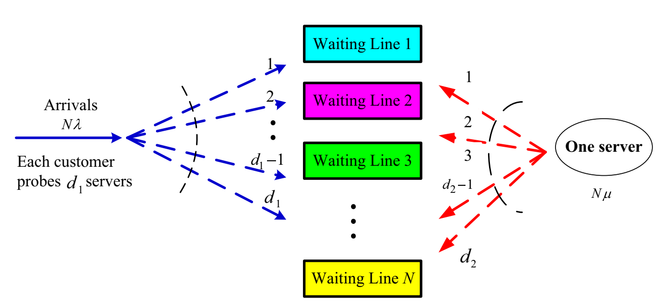

Fig. 1. A physical illustration of the supermarket model with impatient customers.