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Optimal shift partitioning of pharmacies

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ABSTRACT

The pharmacy service requires that some pharmacies are always available and shifts have to be organized: a shift corresponds to a subset of pharmacies that must be open 24 hours a day on a particular week. Under the requirement that each pharmacy belongs to exactly one shift and the assumption that users minimize the distance to the closest open pharmacy during each shift, we want to determine a partition of the pharmacies into a given number of shifts, such that the total distance covered by users is minimized. It may be also required that shift cardinalities are balanced. We discuss different versions and the related computational complexity, showing that the problem is NP-hard in general. A set packing formulation is presented and solved by branch-and-price, together with a fast solution technique based on a tabu search. They have been applied to real and random instances showing that (i) the set packing formulation is very tight and often exhibits no integrality gap; (ii) the branch-and-price solves problems of practical relevance to optimality in a reasonable amount of time (order of minutes); (iii) the tabu search finds optimal or near-optimal solutions in order of seconds.

1. Introduction

Pharmacies in Italy are privately owned and managed. However, due to the particular service they have to provide, pharmacies are subject to special rules that usual shops do not have to obey. For instance, opening a new pharmacy can be allowed only if the ratio pharmacy/population is below a certain threshold and if a new pharmacy can be opened, municipalities are requested to suggest the most convenient location for the community. Moreover, people must be granted that some pharmacies, not too far away, are always available for service, day and night and also on holidays.

To meet this request, pharmacies on large territorial units mutually agree to establish shifts of duty on a rotational basis. A shift lasts a week and during this week the pharmacy must be open 24 hours (although for safety reasons they are locked at night and let customers in only on motivated request). Once all pharmacies have carried out their duty the same shift pattern is repeated.

Essentially the problem is to decide which pharmacies have to carry out their duty in the same week with the idea that these pharmacies must be sufficiently spread over the territory. As already remarked, the pharmacy locations are fixed and cannot be changed. The number of shifts is also usually fixed in advance, even if in principle it could change. Clearly a higher number of shifts correspond to a lighter burden for the pharmacies but to a poorer service to the community.

In mathematical terms one has to partition the set of pharmacies into a fixed number of subsets such that a suitable indicator is minimized, taking into account geographical distances and population sizes. Although facility location problems have been the subject of a very large literature, this particular problem, in which locations are already fixed but we have to decide which facilities to open in each shift, seems not yet investigated. In a companion paper [2] a special version of this mathematical problem restricted to graphs that are trees has been investigated showing that the problem is polynomial and providing an algorithm for its solution. We show in this paper that the mathematical problem is in general NP-hard. For general questions related to computational complexity the reader is referred for instance to [6].

A problem related to pharmacies is dealt with in [11], where the duty lasts only one day and there is no partition of the pharmacies since the frequency at which a pharmacy must be open depends on the local population. Furthermore they have to take care that, due to the different Turkish rules, pharmacies can be opened and closed at any time and without any restriction on the sites.

The paper is organized as follows. In Section 2 we provide a mathematical definition of the problem. Then in Section 3 we investigate the computational complexity of some versions of the problem. In Section 4 we present a set packing formulation with exponentially many variables. A branch-and-price approach for this formulation is presented in Section 5, where the pricing subproblem is modeled as a p-median problem with side constraints. In Section 5.4

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we also present a fast tabu search heuristic to obtain good solutions used as upper bound in the branch-and-price search. A compact Mixed Integer Linear Programming (MILP) model is presented in Section 6. In Section 7 we show a comprehensive set of computational results for real case instances and for random artificial instances. In these tests we compare the two exact methods and the heuristic. Finally we provide some remarks in Section 8.

2. Problem statement

Let F be the set of facilities, i.e., pharmacies and let $n = |F|$. Let C be the set of locations of customers. In each location $i \in C$ there is a population of p_i customers. For each pair $i \in C$, $j \in F$, let d_{ij} be the distance between i and j. For every subset $J \subset F$, let

$$d_i(J) := \min_{j \in J} d_{ij}$$

be the distance of customers in i from the set J.

During a shift some pharmacies must be open. Each pharmacy is open in exactly one shift. Once all pharmacies have completed their duties, the same set of shifts is repeated cyclically. Note that this set of shifts must be a partition of the n pharmacies. Let us call any set of shifts satisfying this partition requirement a *shift partition*. The number of shifts is a fixed number H which is agreed beforehand by all pharmacies. Although not strictly necessary, it is usually required that the shifts are balanced. In this case the number of pharmacies in each shift is either $\lfloor n/H \rfloor$ or $\lceil n/H \rceil$.

We use as an indicator of the quality of a shift partition $J_1, J_2, \ldots, J_H$ the total distance traveled by the customers, assuming that (i) each customer goes to the closest pharmacy, and (ii) on the average every customer goes to a pharmacy the same number of times in each shift. Assumption (i) is typical in location analysis. Furthermore, due to the type of service pharmacies provide, and taking into account that we are specially interested in the night and holiday service, assumption (i) is typically met in practice. In any case, if a customer prefers going to a distant pharmacy, this extra distance is the choice of the customer and should not be 'charged' to the shift quality. Hence it is natural that an indicator takes into account the distance $d_i(J)$ as defined above. As for assumption (ii), it may be difficult (and beyond the scope of this paper) to know in advance when and if customers will go to pharmacies. Hence we assume that all population behaves in the same way and we consider the simple and equivalent case such that a single customer goes exactly once to a pharmacy in each shift. Observe that, if more specific information on customer behavior is available, it can be used to properly weight the population size p_i. Under these assumptions, the distance associated with a shift J is

$$d(J) = \sum_{i \in C} p_i d_i(J)$$

and the total distance traveled by customers, i.e., the cost of the shift partition, may be expressed as

$$\sum_{h \in [H]} d(J_h) \quad (1)$$

(where $[H] := \{1, \ldots, H\}$). Clearly we want to minimize (1) for all feasible shift partitions. From now on we use the more general term *facility* instead of pharmacy.

We observe that every customer has to travel at least the sum of the distances to the H closest facilities, no matter what the shift partition is. Let Δ_i be the sum of the distances from location i to its H closest facilities. We call *utopian optimum* the quantity $\sum_{i \in C} p_i \Delta_i$. Every shift partition cost is lower bounded by the utopian optimum.

There are instances for which the optimal solution is the utopian optimum. It has been proved in [2] that if all customer locations and all facilities are the vertices of a tree (or equivalently of a graph with tree metrics) then there exists a partition meeting the utopian optimum and therefore it must be optimal. The tree structure is fundamental for this result. A simple example showing that the utopian optimum and the optimum cost can be different is a circuit with four vertices, three shifts, unit edge lengths and unit populations. Its utopian optimum is equal to 8 (each vertex-customer has as closest vertices-facilities itself, the vertex at right and the vertex at left, for a total distance of 2, which multiplied by four vertices-customers gives 8). However, it is easy to see that in each shift partition two customers have distance 2 but the other two have distance 3 for a total of 10.

We observe also that minimizing an indicator like (1) does not prevent some $d_i(J_h)$ to be too large in the optimal shift partition with respect to some acceptance threshold. Note that, in at least one shift, each customer has to travel to the $H-$th closest facility whose distance to location i we denote by D_i, i.e., $\max_{h \in [H]} d_i(J_h) \geq D_i$ always hold. Hence by 'too large' we mean a value $d_i(J_h)$ much higher than D_i. In this case we may want to bound $d_i(J_h)$: we will come back to this point later.

3. Computational complexity results

In presenting the computational complexity results we consider the case of unit populations for the sake of simplicity. All results but one will show NP-hardness for the problems, and this clearly implies NP-hardness also for the problems with generic populations. The only polynomiality result remains valid also by extending the problem to generic populations. Let us formally define the problems.

LOCATION PARTITIONING: Given a set F of facilities and a set C of customers, distances $d_{ij} \geq 0$, $i \in C$, $j \in F$, an integer $H \leq n$, a number K, is there a partition of F into subsets $J_1, J_2, \ldots, J_H$, such that $\sum_{h \in [H]} \sum_{i \in C} \min_{j \in J_h} d_{ij} \leq K$?

BALANCED LOCATION PARTITIONING: Given a set F of facilities and a set C of customers, distances $d_{ij} \geq 0$, $i \in C$, $j \in F$, an integer $H \leq n$, a number K, is there a partition of F into subsets $J_1, J_2, \ldots, J_H$, such that $|J_h| \in \{\lfloor n/H \rfloor, \lceil n/H \rceil\}$ for any h, and $\sum_{h \in [H]} \sum_{i \in C} \min_{j \in J_h} d_{ij} \leq K$?

Both problems can be further specialized according to the assumptions we make on distances d_{ij} and on customer and facility sets. We consider the following three versions of the problems in the order of decreasing generality:

(a) distances d_{ij} are nonnegative;
(b) customer locations and facilities are two subsets (not necessarily disjoint) of the vertices of a given graph and the d_{ij} are the shortest distances with respect to nonnegative edge lengths of the graph;
(c) as in (b) but the two subsets coincide and include all the vertices of the graph.

Theorem 1. *LOCATION PARTITIONING is NP-complete for $H \geq 3$ and for all three versions.*

Proof. First note that for version (a) the instance size is $\Omega(n|C|)$ due to the list of distances d_{ij} and for versions (b) and (c) is $\Omega(n)$ due to the list of edge lengths. Since the solution size is $O(n \log n)$ and computing $\sum_{h \in [H]} \sum_{i \in C} \min_{j \in J_h} d_{ij}$ takes time $O(n|C|)$, checking a yes-instance is polynomial with respect to the input length. Hence the problem is in NP.

We prove the NP-completeness via a transformation from DOMATIC NUMBER [4,6, p. 190]. We recall that, given a graph $G = (V, E)$, DOMATIC NUMBER asks whether there exists a partition of V into at least H dominating subsets. We also recall that a subset of vertices is dominating if each vertex of G is either in the subset or is adjacent to some vertex in the subset. DOMATIC NUMBER is NP-complete for $H \geq 3$.