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The production routing problem: A review of formulations and solution algorithms

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ABSTRACT

The production routing problem (PRP) combines the lot-sizing problem and the vehicle routing problem, two classical problems that have been extensively studied for more than half a century. The PRP is solved in an attempt to jointly optimize production, inventory, distribution and routing decisions and is thus a generalization of the inventory routing problem (IRP). Although the PRP has a complicated structure, there has been a growing interest in this problem during the past decade in both academia and industry. This article provides a comprehensive review of various solution techniques that have been proposed to solve the PRP. We attempt to provide an in-depth summary and discussion of different formulation schemes and of algorithmic and computational issues. Finally, we point out interesting research directions for further developments in production routing.

1. Introduction

In a typical supply chain which consists of sequential activities of production, storage and distribution, each individual process is often planned and optimized using predetermined decisions from its preceding activities. For example, a production planner makes production lot-sizing decisions in order to minimize production and inventory costs at the production facility. The planned lot-sizing decisions are then used as inputs in subsequent steps of distribution planning. Since the decisions are limited by the plan of the former process, the benefits of coordination in the planning process have been left behind. An integrated supply chain operational planning system is a tool that is used to jointly optimize several planning decisions thereby capturing the additional benefits of coordination between sequential activities in the chain. In recent years, many companies, such as Kellogg [21] and Frito-Lay [25], have set up integrated planning systems and achieved multi-million cost savings. The key to success is an application that is not only able to produce solutions with minimal costs, but that can also be used in an effective and timely manner.

The production routing problem (PRP) is an integrated operational planning application that jointly optimizes production, inventory, distribution and routing decisions. It is of practical relevance in a Vendor Managed Inventory (VMI) approach, in which the supplier monitors the inventory at retailers and also decides on the replenishment policy for each retailer. The supplier acts as the central decision maker who solves an integrated supply chain planning problem. The advantage of a VMI policy with respect to the traditional retailer managed inventory system lies in a more overall efficient resource utilization. The PRP connects two well-known problems, namely the lot-sizing problem (LSP) and the vehicle routing problem (VRP), to produce an optimal solution when considering the total system cost. The PRP is also a generalization of the lot-sizing problem with direct shipment and of the inventory routing problem (IRP). Solving the PRP becomes challenging as it is a combined version of the LSP and VRP and it incorporates the constraints of these two difficult problems. We aim to provide an in-depth review of the PRP, particularly with respect to the formulations and solution algorithms. Different formulation schemes of the PRP are examined. The approaches to compute lower bounds, exact algorithms and heuristics are thoroughly reviewed. We further discuss future research directions.

In the rest of this section, we first provide a brief overview of the three integrated problems. The network representations of these problems in the case of a single supplying facility and multiple customers in a discrete time finite horizon are presented in Fig. 1. Note that the supplying facility can be a *plant* with setup costs and production decisions or a *warehouse* with fixed ordering costs and ordering decisions. In each period, a single or multiple products can be made available at the supplying facility and they are transported to the customers in order to satisfy demands. The products can be stored at the plant or at the customers, thus incurring inventory holding costs.

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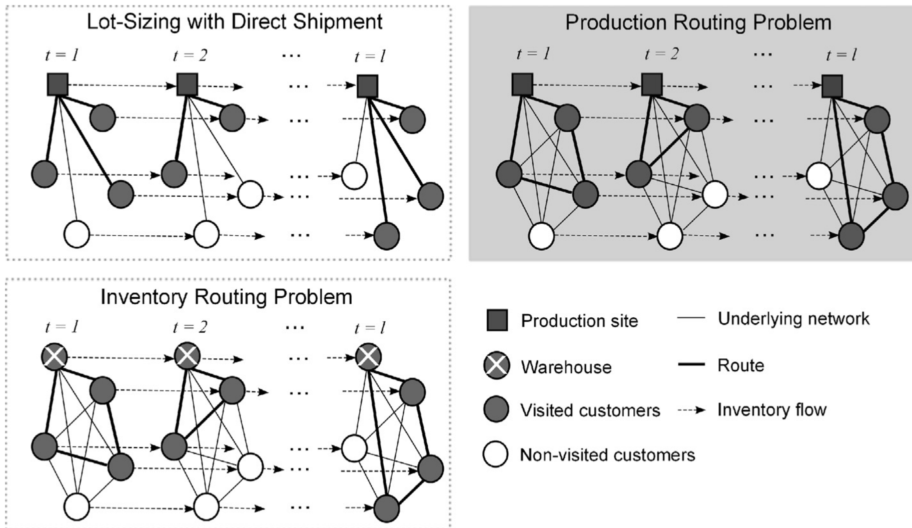

Fig. 1. Network representations of the integrated problems.

1.1. *Integrated lot-sizing with direct shipment*

In this problem, the products are directly transported from the manufacturing plant to the customers. The production, setup, inventory and direct shipment costs are minimized over the planning horizon. This problem typically incorporates various production aspects, e.g., production setup cost and/or setup time, and involves distribution decisions where the fixed and unit costs of delivery are customer specific.

The integrated production and direct shipment distribution planning was studied by several researchers. Most of the studies considered the distribution cost as a fixed cost or a complex cost function. Li et al. [56] focused on the lot-sizing problem with a piecewise linear transportation cost function where the supplier has the option to deliver by direct delivery with truckload (TL) or less-than-truckload (LTL) transportation. They developed a dynamic programming approach to solve the one-product and one-customer problem. Jaruphongsa et al. [50] proposed other dynamic programming algorithms to solve the problem with TL and LTL cost structures. A problem with a special cost structure, where the supplier can get a discount from transportation capacity reservation, was studied by van Norden and van de Velde [79]. A more general piece-wise linear transportation cost function was addressed by Rizk et al. [67]. They decomposed the integrated problem into uncapacitated lot-sizing and time-independent subproblems and applied a Lagrangian relaxation technique to obtain lower bounds. In the more general case of multiple customers, Chand et al. [26] developed a dynamic programming algorithm to solve the problem in which backlogging is allowed. Jaruphongsa and Lee [51] considered the problem with split delivery under time window restrictions and employed dynamic programming algorithms to solve the problem. A special problem of lot-sizing with truckload shipment where transshipments between the customers are allowed was considered by Herer and Tzur [47]. The multi-item problem with one customer was considered by Lee et al. [53]. In the case of uncapacitated production and uncapacitated vehicles, the problem with direct shipments is also known as the one-warehouse multi-retailer problem (OWMR). Federgruen and Tzur [36] considered the OWMR with multiple items and developed a time-partitioning heuristic to solve the problem. Solyalı and Süral [76] proposed a new strong formulation based on the combined transportation and shortest path model to solve the OWMR with a single product. Melo and Wolsey [61] discussed several formulations and proposed hybrid heuristics for the two-level production–transportation problems with capacitated production and vehicles.

There is a link between the lot-sizing problem with truckload cost structure and the classical lot-sizing problem with batch size where the batch quantity is smaller than the maximum production quantity in one period. The truck capacity can be viewed as the fixed batch quantity limit and the cost of dispatching one truck can also be considered as the fixed production cost of one batch. There is also a link between the lot-sizing problem with transshipments and the lot-sizing problem with production substitution where a product can be used to substitute for the demand of another product [48]. The cost of transshipment between customer locations can be viewed as the cost of production substitution.

1.2. *Inventory routing problem (IRP)*

When the routing aspect is included and the production aspect is disregarded, the problem is transformed into the inventory routing problem (IRP). In the IRP, the starting point is a warehouse where there is no production decision as the production quantities made available in each period are typically given. A vehicle can visit more than one customer by travelling along its route. As a generalization of the VRP, which consists of the decisions on delivery quantities and routes to serve customers, the IRP also includes the timing to serve the customers' demands. This makes the problem much more difficult than the classical VRP due to the complex periodic routing and inventory decisions. The IRP is obviously NP-hard since it contains the VRP as a special case [32].

The IRP first appeared in a gas delivery study by Bell et al. [15]. The problem was solved using a Lagrangian relaxation method and was decomposed by time period and by vehicle. Carter et al. [24] and Campbell and Savelsbergh [22] proposed efficient heuristic procedures by decomposing the IRP into an allocation problem (AP) and a vehicle routing problem (VRP). Since the IRP is a complicated combinatorial problem, several metaheuristics, e.g., tabu search [70], genetic algorithm [1], greedy randomized adaptive search procedure (GRASP) [71], hybrid heuristic with combined tabu search and MIPs [8], and adaptive large neighborhood search (ALNS) [31,32], have been proposed. Gaur and Fisher [42] discussed a periodic IRP where the demand pattern is repeated and developed a heuristic to solve the problem.

As mentioned in Andersson et al. [6], few exact algorithms have been proposed to solve the IRP due to its complexity. Notable exceptions include a branch-and-cut procedures to solve the IRP

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