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A decomposition-based heuristic for the multiple-product inventory-routing problem

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ABSTRACT

The inventory-routing problem is an integrated logistics planning problem arising in situations where customers transfer the responsibility for inventory replenishment to the vendor. The vendor must then decide when to visit each customer, how much to deliver and how to sequence customers in vehicle routes. In this paper, we focus on the case where several different products have to be delivered by a fleet of vehicles over a finite and discrete planning horizon. We present a three-phase heuristic based on a decomposition of the decision process of the vendor. In the first phase, replenishment plans are determined by using a Lagrangian-based method. These plans do not specify delivery sequences for the vehicles. The sequencing of the planned deliveries is performed in the second phase in which a simple procedure is employed to construct vehicle routes. The third phase incorporates planning and routing decisions into a mixed-integer linear programming model aimed at finding a good solution to the integrated problem. Computational experiments show that our heuristic is effective on instances with up to 50 customers and 5 products.

1. Introduction

In the last decades, the availability of sophisticated information technologies has led to the development of several new business models in supply chain management. Among these, Vendor-Managed Inventory (VMI) represents a modern paradigm in which a vendor (or supplier) has the responsibility for controlling the inventory of the customers, on the basis of a day-to-day monitoring of their stocks, and for deciding the replenishment policies. The potential benefits from the application of a VMI strategy are numerous. Mainly, the vendor can increase the level of service and reduce the distribution costs through effective vehicle utilization, while the customers can devote less resources to manage their inventories and to place orders [20,32]. The vendor adopting a VMI system must solve an integrated inventory-routing problem (IRP). The objective of this problem is to minimize the overall logistic cost for supplying customers over a finite planning horizon, while avoiding stockouts at any of the customers. Three types of decisions must be made: (i) when to service a customer, (ii) how much to deliver to a customer, (iii) which delivery routes to use for the service. The scientific literature is rich with contributions related to inventory control problems as well as node, arc and general routing problems tackled in a separate way (see, e.g., [18,27]). Integrated problems have been studied less extensively. In the following, we discuss some of the IRPs addressed in the scientific literature. For a more complete overview, we refer the reader to reviews and tutorials [2,9–11,14,23].

Many IRP contributions were motivated by real applications. For instance, Campbell and Savelsbergh [12] were inspired by work done with Praxair, an international industrial gases company. Praxair's production activity involves taking air and separating it into its components, such as oxygen, hydrogen, nitrogen, and argon; the gases are transported to the customers in their liquid form by using trucks. Other common applications arise in maritime logistics, where compartmented ships are used for replenishment. Although maritime IRPs are similar to their road transportation counterpart, they also involve several special features, as discussed in [26].

A common criterion to classify the existing literature is the length of the planning horizon: single-period, multi-period or infinite horizon. Federgruen and Zipkin [17] approached the IRP as a single-day problem. Specifically, they treated a single-period problem characterized by a plant, with limited amount of available inventory, and customers with random demands. Chien et al. [13] tackled a single-period IRP with a deterministic demand at the customers. They formulated the problem as a mixed-integer program and developed a Lagrangian-based procedure to generate heuristic

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solutions. With respect to the multi-period version, most commonly studied IRPs are problems where a single product is delivered through a single vehicle, in a deterministic VMI setting. Exact algorithms (e.g., [5,29]) and heuristics (e.g., [4]) are available for this case. Some authors focused on IRPs with an infinite horizon. For instance, Anily and Federgruen [3] considered distribution systems with a central warehouse and many geographically dispersed customers. The warehouse faces external demand for a single product occurring at constant, deterministic but customer-specific rates. The authors determined feasible replenishment strategies while minimizing (infinite horizon) long-run average transportation and inventory costs.

In this paper, we deal with a multi-period problem in which multiple products are resupplied to a set of customers from a common vendor and deterministic demands occur at the customers for each product. Starting from [3], Viswanathan and Mathur [31] studied a system distributing different products and presented a heuristic algorithm generating a stationary nested joint replenishment policy to coordinate routing and inventory decisions. A computational study was performed on randomly generated instances of moderate size and restricted to the case of a single product in order to carry out comparisons. Popović et al. [25] tackled a multi-product IRP with a focus on fuel delivery. They presented a mathematical formulation of the problem and a variable neighborhood search heuristic. Moin et al. [24] and Mjirda et al. [22] addressed a multi-product IRP where vehicle routes collect products at supplier locations to meet the demand of an assembly plant. In their problem, each supplier provides a unique product and can be visited by more than one vehicle in the same time period. A similar replenishment problem was also studied by Sindhuchao et al. [28]. Most of the aforementioned contributions were inspired by an applicative context. The first general contribution to the IRP with multiple vehicles and multiple products can be considered to be that of Coelho and Laporte [15] who proposed mixed-integer linear programs for a multi-product IRP and for its counterpart with consistency features. The authors developed a branch-and-cut algorithm to obtain exact solutions. Very recently, Coelho and Laporte [16] also introduced a branch-and-cut algorithm for a further generalization of the IRP where vehicles have several compartments for the different products being delivered.

Our focus is on a multi-product multi-vehicle IRP (MMIRP) under a maximum-level policy, where in each period and for each product the amount delivered to a customer cannot exceed a given maximum storage level. In order to solve this problem, we decompose the decision process into planning and routing subprocesses, and try to integrate the two parts with the help of a mixed-integer linear programming model. More precisely, we propose a three-phase heuristic algorithm. In the first phase, delivery plans are determined by using a Lagrangian-based method. The use of Lagrangian relaxation and Lagrangian heuristics is frequent in the IRP literature (see, e.g., [30,33,34]). In the second phase, delivery sequences for the vehicles are specified. In particular, a simple heuristic procedure is employed to construct the routes. In our context, split delivery (see [6,7]) is allowed. The third phase utilizes mixed-integer linear programming to improve the solution in an integrated way.

The rest of the paper is organized as follows. In Section 2, we formally define the MMIRP and discuss the solution strategy. The first, second and third phases are covered in greater depth in Sections 3, 4 and 5, respectively. Section 6 presents computational results, while Section 7 highlights general conclusions.

2. Notation and algorithm overview

The MMIRP can be modelled on an undirected graph $G=(V,E)$ where $V=\{0,1,\ldots,n\}$ is a set of nodes and $E=\{(i,j): i,j\in V, i<j\}$ is a set of edges. A non-negative cost is associated with each edge. A set K of homogeneous vehicles with capacity Q is available at the vendor, represented by node 0, which organizes the deliveries to the customers in the set $N=V\backslash\{0\}$ over a finite and discrete planning horizon $T=\{1,\ldots,H\}$. To evaluate the impact of the decisions made at the end of the planning horizon, the set T is augmented with an additional period $H+1$ and we refer to $T'=\{1,\ldots,H,H+1\}$ as the extended planning horizon. A set P_i of different products is associated with each customer i. We assume here that the deliveries and the consumption rates are continuous quantities. For this reason, not all applications can be modelled with the proposed methodology. The deliveries are organized by the vendor on the basis of the maximum inventory level U_{pi} and the initial inventory level I^0_{pi} $(i\in N, p\in P_i)$. Let μ^t_{pi} be the quantity of product p consumed at customer i in period t. We can realistically suppose that $\mu^t_{pi}\le U_{pi}$ for every $t\in T$, $i\in N$, and $p\in P_i$. Let h_{pi} be the unit inventory holding cost of product p at customer i. The MMIRP aims to find a replenishment plan that minimizes the total inventory and routing costs while avoiding stockouts for every product at every customer over the planning horizon.

The problem that we address is similar to that studied by Coelho and Laporte [15] but we assume here that each customer has a maximum inventory level for each product instead of a shared capacity for all products. This problem arises, in particular, when customers have dedicated storage locations for each specific product and cannot mix them. We also assume that the supplier has an infinite capacity and that there are no inventory holding costs at the supplier. We refer the reader to their manuscript for a complete formulation of the inventory-routing problem in a multi-product multi-vehicle system.

Our solution strategy combines several ideas from the literature (in particular [12]) and adapts them using insights from practical industrial projects on VMI. More precisely, we decompose the decision process into two subprocesses, i.e. planning and routing, and heuristically solve them. A *feedback model* is also used to improve the planning with information obtained from the routing subprocess. Therefore, our strategy consists of three phases. In the first one, named as *planning phase*, planning solutions are obtained by using a Lagrangian-based method. In the second phase, named as *routing phase*, routing solutions (i.e., sets of multiple routes consistent with the planning solutions) are obtained by using a simple heuristic algorithm and allowing split deliveries. It is possible to identify an MMIRP solution at the end of the second phase. In the final phase, named as *reoptimization phase*, a feedback model is applied in order to improve this solution.

3. Planning phase

In the planning phase, we use a mixed-integer programming model to determine good replenishment plans. These plans specify which customers to serve in every period of the planning horizon and how much to deliver each product on each occasion. Section 3.1 explains how the problem can be formulated, while Section 3.2 presents a subgradient algorithm to solve the problem.

3.1. Customers' inventory model

The Customers' Inventory Model (CIM) uses the notation introduced in Section 2. Let M be a large constant and g_i the cost for servicing customer i. The CIM can be defined by using the following decision variables. Let d^t_{pi} be the quantity of product p delivered to customer i in period t. Also let y^t_i be a binary variable equal to 1 if customer i is visited in period t, 0 otherwise. Finally, I^t_{pi} represents the inventory level of product p at the beginning of