Computers & Operations Research 41 (2014) 1–11

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

An effective matheuristic for the capacitated total quantity discount problem

Daniele Manerba*, Renata Mansini

Department of Information Engineering, University of Brescia, Italy

ARTICLE INFO

Available online 29 July 2013

Keywords:
Supplier selection
Total Quantity Discount
Matheuristic
ILP refinement

ABSTRACT

New generation algorithms focus on hybrid miscellaneous of exact and heuristic methods. Combining meta-heuristics and exact methods based on mathematical models appears to be a very promising alternative in solving many combinatorial optimization problems. In this paper we introduce a matheuristic method exploiting the optimal solution of mixed integer linear subproblems to solve the complex supplier selection problem of a company that needs to select a subset of suppliers so to minimize purchasing costs while satisfying products demand. Suppliers offer products at given prices and apply discounts according to the total quantity purchased. The problem, known as Total Quantity Discount Problem (TQDP), is strongly $\mathcal{NP}-$hard thus motivating the study of effective and efficient heuristic methods. The proposed solution method is strengthened by the introduction of an Integer Linear Programming (ILP) refinement approach. An extensive computational analysis on a set of benchmark instances available in the literature has shown how the method is efficient and extremely effective allowing us to improve the existing best known solution values.

1. Introduction

Supplier selection problem is usually formalized in terms of two joint decisions concerning which suppliers should be selected and how much should be ordered from each of them. As supplier selection represents a critical task in any organization, the problem has been studied since long (Weber et al. [34]), and in the last few decades has gained increasing attention both in practice and theory mainly fueled by its web counterpart (see e.g. [22]).

Real procurement problems are usually complicated by side constraints taking into account both qualitative and quantitative factors. The introduction of discount policies and/or traveling costs are among the most complicating features of real procurement settings. In this paper we deal with a supplier selection problem where suppliers apply volume discounts depending on the total quantity ordered. The problem can be described as follows. Let $S:=\{1,\ldots,n\}$ be a set of suppliers, and let $K:=\{1,\ldots,m\}$ be a set of products. Each product $k\in K$ can be purchased in a subset $S_k\subseteq S$ of suppliers at a positive price p_{ik}, potentially different for each supplier $i\in S_k$. For each product $k\in K$ a positive integer demand d_k is defined, and for each product $k\in K$ and each supplier $i\in S_k$ a positive quantity availability q_{ik} is specified, such that $\sum_{i\in S_k} q_{ik}\geq d_k$. Each supplier $i\in S$ defines a set $R_i=\{1,\ldots,r_i\}$ of r_i consecutive and non-overlapping discount intervals $[l_i^r, u_i^r]$, where l_i^r and u_i^r are the minimum and the maximum number of product units, respectively, that have to be purchased to lay in interval r of supplier i (w.l.o.g. we assume that $\sum_{k\in K} q_{ik}\leq u_i^{r_i}$ and $l_i^1=0$ for each $i\in S$). A non-negative discount rate δ_i^r is associated to each interval $r\in R_i$ such that $\delta_i^{r+1}\geq\delta_i^r$, $r=1,\ldots,r_i-1$. The interval in which the total quantity purchased lies determines the discount applied by the supplier to the total purchase cost (*total quantity discount policy*). For the sake of simplicity we can convert the discount rates into non-increasing unit prices $p_{ik}^r>0$, i.e. $p_{ik}^{r+1}\leq p_{ik}^r$, $i\in S_k$, $r=1,\ldots,r_i-1$. The Capacitated Total Quantity Discount Problem (CTQDP) looks for a subset of suppliers so that the total demand is satisfied at a minimum purchasing cost.

In the classical Total Quantity Discount Problem (TQDP) suppliers are assumed to have unlimited availability for offered products. The problem is introduced in Goossens et al. [18] where the authors show its $\mathcal{NP}-$completeness by reduction from the 3-Dimensional Matching Problem (proof is done assuming $l_i^1=0$ for all the suppliers) and demonstrate that it cannot be solved by a polynomial-time approximation algorithm with a constant ratio (unless $\mathcal{P}=\mathcal{NP}$). They also study four variants of TQDP that involve market share constraints, the possibility of buying more than products demand, a limited number of winning suppliers and multi-period scenarios. Finally, they propose a branch-and-bound algorithm based on a min-cost flow problem formulation on

* Corresponding author. Tel.: +39 0303715935.
E-mail addresses: daniele.manerba@ing.unibs.it (D. Manerba), renata.mansini@ing.unibs.it (R. Mansini).

0305-0548/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.cor.2013.07.019

instances with up to 50 suppliers, 5 discount intervals and 100 products. Nowadays, all these instances can be solved to optimality by a standard commercial solver in a matter of seconds, ceasing to be challenging.

More recently, Manerba and Mansini [24] develop a branch-and-cut approach for the Capacitated TQDP exploiting simple polyhedral results. The authors define a new set of hard-to-solve benchmark problems including up to 100 suppliers, 5 discount intervals and 500 products. Instances are divided into two classes and can be downloaded at the page http://www.ing.unibs.it/~orgroup/instances.html. The largest ones of these instances have not been solved to optimality yet.

In the present paper we use a straightforward heuristic that takes advantage of the information provided by the linear programming (LP) relaxation of the original problem to construct an initial solution, and study a stand-alone matheuristic for the CTQDP based on the optimal solution of mixed integer subproblems. The framework of the algorithm is similar to a Variable Neighborhood Decomposition Search (VNDS) as described in Hansen et al. [20]. Iteratively, some features of a current feasible solution are kept fixed and a Mixed Integer Linear Programming (MILP) subproblem is formulated and solved exactly. Computing optimal solutions, although computationally intractable, can become a valid tool provided that the subproblem solved is reasonably small. The proposed method is named HELP2 since it extends an embrional procedure called HELP (Heuristic Enhancement from LP) proposed in Manerba and Mansini [24]. We also introduce an ex post improvement procedure based on an Integer Linear Programming (ILP) refinement that enhances the performance of the basic algorithm. Heuristics behavior has been tested on the benchmark instances described above.

The paper provides the following contributions. HELP2 and its variant including ILP refinement represent new matheuristics for the CTQDP. Computational experiments show that the methods perform effectively for a variety of problem instances, including those taken from a real world application. In particular, HELP2 finds the optimal solution in about 48% of benchmark instances and in the remaining ones the total average error is equal to 0.16% for a class of instances and to 1.18% for the other one, whereas HELP2 with the ILP refinement further reduces the total average error to 0.56% on the most difficult instances improving 10 best known values. The heuristics owe their strength to simple basic ideas that could be generalized also to other MILP problems presenting a similar combinatorial structure. Finally, from a practical point of view, while the importance of the CTQDP is widely recognized, no solution methods can be found in the literature to tackle this real problem. To the best of our knowledge, HELP2 represents the first stand-alone heuristic methods for the CTQDP and thus provides the first reliable tool for decision makers handling similar procurement problems.

The paper is organized as follows. Section 2 is devoted to the literature review, focused either on existing algorithms for procurement problems or on general matheuristics. In Section 3 we describe a mathematical model for the problem, including useful valid inequalities added to strengthen the LP formulation. In Section 4 the procedure used to find an initial feasible solution as well as HELP2 and its extension including the ILP refinement (called HELP2+) is analyzed in detail. Extensive computational results are described in Section 5. Finally, some conclusions are drawn in Section 6.

2. Literature review

We first review solution approaches proposed for procurement problems, then we discuss contributions on hybrid methodologies (matheuristics) for more general problems.

In the literature, supplier selection problems are mainly faced in terms of quality criteria (Dickson [12] identifies 23 different critical factors for selecting suppliers). In the last few years a part of the specialized literature was focused on mathematical modeling and heuristic approaches for procurement problems with deterministic or stochastic demand. Some of these works take into account discount policies, but none of them introduces a solution approach for the CTQDP. Benton [7] proposes a Lagrangian relaxation based heuristic to solve a supplier selection problem with multiple item, multiple supplier and quantity limitation. Another Lagrangian relaxation based heuristic is developed in Güder et al. [19] to solve a multiple item material cost minimization problem with incremental discount offered by a single supplier. Chauhan and Proth [9] propose a heuristic solution for a variant with concave purchasing cost and thresholds on ordering quantities in a multi-supplier system in the case of single product and in the multi-item one. Burke et al. [8] introduce a simple heuristic method for solving a quantity allocation decision in a multiple supplier setting characterized by three different pricing schemes (linear discounts, incremental units discounts and all units discounts), whereas Awasthi et al. [6] propose a method for a problem with limitation on minimum and maximum order sizes in a multi-source system.

Procurement problems real setting may be complicated by the inclusion of transportation costs either as truckload shipping costs or as cost of visiting suppliers in one or more tours. A classical and well-studied procurement problem involving both traveling and purchasing costs is the Travelling Purchaser Problem (TPP). Several heuristic and exact methods have been proposed for the problem. In Voss [33] the author investigates two dynamic tabu search strategies, whereas in Riera-Ledesma and Salazar-González [29] the authors propose a local search heuristic based on node chains removal. More recently, in Angelelli et al. [4,5] the authors analyze a dynamic extension of the TPP where quantities offered by the suppliers are assumed to decrease over time. Two groups of heuristics are described and compared. The first group consists of simplified approaches based on greedy criteria, the second one includes heuristics based on a look-ahead approach taking future prediction into account. Mansini and Tocchella [27] analyze a version of the TPP where total purchasing costs do not exceed a predefined threshold. The authors provide an enhanced local search heuristic and a Variable Neighborhood Search (VNS) tested in a multi-start variant. Finally, in Mansini et al. [26] the TQD procurement problem is extended to include truckload shipping costs. The authors develop integer programming based heuristics to solve the problem and demonstrate their efficacy on a large set of randomly generated instances.

Over the last few years, the best results found for many practical or academic optimization problems have been obtained using hybrid algorithms. A hybrid method can be thought as an approach combining the principles of different solution methods, usually represented by meta-heuristics (see [31] for a taxonomy). Among the hybrid methods, matheuristics are receiving an increasing attention from research community, thanks also to the great advance in Integer Programming solvers. Matheuristics combine successfully meta-heuristics and mathematical programming techniques to solve complex combinatorial optimization problems (see [25]). Different forms of cooperation between heuristics and exact techniques can be figured out. Some methods use exact algorithms to explore large neighborhoods in local search algorithms. Other methods perform several runs of a local search and exploit information in high quality solutions to define smaller problems that can be easily solved with exact algorithms. Finally, some matheuristics use information from relaxation of integer programming problems to guide local search. All methods described below belong to one of these categories.

Glover et al. [17] investigate whether principles of a Tabu Search algorithm can be embedded into a branch-and-bound structure to solve a graph coloring problem. Haouari and Ladhari [21] analyze a