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A redistricting problem applied to meter reading in power distribution networks

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ABSTRACT

The capacitated redistricting problem (CRP) has the objective to redefine, under a given criterion, an initial set of districts of an urban area represented by a geographic network. Each node in the network has different types of demands and each district has a limited capacity. Real-world applications consider more than one criteria in the design of the districts, leading to a multicriteria CRP (MCRP). Examples are found in political districting, sales design, street sweeping, garbage collection and mail delivery. This work addresses the MCRP applied to power meter reading and two criteria are considered: compactness and homogeneity of districts. The proposed solution framework is based on a *greedy randomized adaptive search procedure* and multicriteria scalarization techniques to approximate the Pareto frontier. The computational experiments show the effectiveness of the method for a set of randomly generated networks and for a real-world network extracted from the city of São Paulo.

1. Introduction

The objective of districting problems (DP) is to cluster small territorial units into contiguous and non-overlapping districts, given an objective function and possible side constraints. A common application of DP is political districting, in which a geographic area has to be partitioned into electoral districts, each one represented by a member of parliament [1–3]. Public transportation network pricing systems [4], commercial territory design [5], salesman working zones design [6–8] and definition of areas for manufactured and consumer goods [9] are also DP applications.

Some methods that have been proposed to solve the DP are tabu search [10,11], evolutionary algorithm [4], basic descent algorithm, simulated annealing, old bachelor acceptance algorithm [11] and a greedy randomized adaptive search procedure (GRASP) with adaptive memory programming (AMP) [12].

The capacitated districting problem (CDP) is a generalization of DP in which districts have a limited capacity. Ríos-Mercado and Fernandez [5] deal with a commercial territory design problem in which the districts must be balanced with respect to a set of commercial activities. To solve this problem, they propose a reactive GRASP with the objective of minimizing the largest euclidean distance between two nodes of the same district. An extension of this approach also considers a routing budget side constraint [13].

The multicriteria capacitated districting problem (MCDP) is a CDP with more than one objective in the design of districts. Surveys on multiobjective optimization theory, Pareto optimality and techniques are referred [14–18]. Salazar-Aguilar et al. [19] use a scatter search metaheuristic to solve an MCRP with two criteria: district dispersion and balance of the number of customers.

A more general problem is the multicriteria capacitated redistricting problem (MCRP). The redistricting implies the existence of an original set of districts in the geographic network under study. This work applies the MCRP to reassign power utility customers into new districts. The expansion of cities with new developments, people migration, and uneven changes of power demand in the suburbs are examples of forces that pressure the redefinition of districts. Each district refers to the working zone of a group of meter readers that perform readings of power consumption from the customers of that same district. The readings are performed *in situ* and feed the monthly invoice sent to each customer.

This paper proposes a solution framework for the MCRP based on a GRASP metaheuristic and a multicriteria scalarization technique. The approximate Pareto frontier is obtained iteratively by solving mono-objective problems in which the objective function is a weighted sum expression of the two criteria under consideration.

A post-optimization problem, not addressed in this paper, is to define the tours to perform the readings in each district [20–23]. Given that effective tours require compact and balanced districts,

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utilities may incur in high operational costs whether districts are too disperse or have uneven workloads. This is due to the required number of readers, which could vary substantially from one district to another.

The redistricting performed in this work is based on two criteria: (i) compactness, aiming low dispersion of districts, and (ii) workload balance, to require the same number of meter readers per district. In addition, a threshold is defined restraining the number of customers that are assigned to districts different from their original ones.

Computational tests were performed using a set of randomly generated networks and a real-world six million customers network extracted from the São Paulo metropolis, Brazil. The results show that the proposed GRASP provides high-quality efficient solutions within acceptable execution times.

The paper is organized as follows. Section 2 introduces the MCRP and proposes an integer linear programming model. Section 3 describes the MCRP application to meter reading. The proposed GRASP metaheuristic is presented in Section 5. Section 6 gives the computational experiments and the analysis of the results. Conclusions and final remarks follow in Section 7.

2. Multicriteria capacitated redistricting problem model

A mathematical model derived from the multicriteria model proposed in [5] is used to address the MRCP considered in this work. The original model referred to a commercial territory design problem with no redistricting.

The MCRP is defined on a connected undirected graph $G(V,E)$ where V is the set of n nodes and E is the set of m edges. Each node has a set A of associated activities and w_i^a is the demand of activity $a \in A$ from node i. The graph is Euclidean, and the distance of each pair of nodes i and j is given by d_{ij}. The nodes adjacent to i is given by N^i. The objective of the MCRP is to find a partition of the node set V into P districts ($|V| \geq P$), $\{V_1, V_2, \dots, V_P\}$, with respect to two criteria: district compactness and workload homogeneity. A set of artificial nodes $V_0 = \{1, 2, \dots, P\}$ represents the centers of the districts. The demand target of activity a for all districts is given by $\mu^a = \sum_{i=1}^{n} w_i^a / P$. Parameter l_{ki} represents the number of customers in node i originally assigned to district k. The maximum number of customers that can change from their original district is given by L.

The MCRP can be formulated as the following integer linear programming problem:

$$x_{ki} = \begin{cases} 1, & \text{if node } i \text{ is assigned to district } k; \ i \in V, k \in V_0, \\ 0, & \text{otherwise} \end{cases}.$$

$$y_{ijk} = \begin{cases} 1, & \text{if nodes } i \text{ and } j \text{ are both assigned to district } k; \ i,j \in V, k \in V_0, \\ 0, & \text{otherwise} \end{cases}.$$

$$\text{(MCRP)} \min \sum_{k=1}^{P} \max_{i,j \in V} d_{ij} y_{ijk} \tag{1}$$

$$\min \sum_{k=1}^{P} \sum_{a \in A} \sum_{i=1}^{n} |w_i^a x_{ki} - \mu^a| \tag{2}$$

$$\text{s.t.} \sum_{k=1}^{P} x_{ki} = 1 \quad i \in V \tag{3}$$

$$\sum_{k=1}^{P} \sum_{i=1}^{n} l_{ki}(1 - x_{ki}) \leq L \tag{4}$$

$$\sum_{i \in \cup_{v \in D} N^v \setminus D} x_{ki} - \sum_{i \in D} x_{ki} \geq 1 - |D| \quad k \in V_0, \ D \subset V \tag{5}$$

$$y_{ijk} \leq x_{ik} \quad i,j \in V, \ k \in V_0 \tag{6}$$

$$y_{ijk} \leq x_{jk} \quad i,j \in V, \ k \in V_0 \tag{7}$$

$$y_{ijk} \geq x_{ik} + x_{jk} - 1 \quad i,j \in V, \ k \in V_0 \tag{8}$$

$$x_{ki}, y_{ijk} \in \{0,1\} \quad i,j \in V, \ k \in V_0 \tag{9}$$

The first objective function (1) minimizes the sum of the greatest distance from each district (compactness). The second objective function (2) minimizes the sum of demand deviations from the target values (homogeneity). Constraint (3) assures that each node must be assigned to a single district. Constraint (4) limits the number of costumers that can change district. If any subset of non-artificial nodes $D \subset V$ contains a node that belongs to district k, then constraint (5) ensures there will be another node in the neighborhood of D that also belongs to district k (more details in [24]). Constraints (6) and (8) ensure that if nodes i and j are both assigned to a same district ($x_{ik} = x_{jk} = 1$) then $y_{ijk} = 1$. Variable domains are given in (9).

3. Application to power distribution networks

The MCRP application in this work is a redistricting problem for power distribution utilities. The utility concession area is partitioned into districts used for meter reading. Over time, the districts become obsolete and a redistricting problem arises. This problem is to update the current partition into P districts, where P is an arbitrary integer that can be greater, equal or less than the current number of districts.

The graph representation of the geographic area is done by associating a node to every street corner and an edge to each street segment between two corners (Fig. 1). Each edge has two demands w^a: *reading time* ($a=1$) and *number of customers* (or number of electric meters) ($a=2$). The reading time is how long a meter reader takes to perform all the metering on a street segment.

A district k can be considered as the induced subgraph of a subset $V_k \subseteq V$ of nodes. A district is considered feasible when the corresponding induced subgraph is connected. A feasible solution is formed by a disjointed set of non-overlapping feasible districts containing all nodes of V.

To perform the graph partitioning, three criteria are considered:

(1) *Compactness*. The geographic shape of the new districts should be as compact as possible, since elongated and tortuous districts tend to hamper the definition of good tours. The compactness is represented by the first objective function (1) in the MCRP model.
(2) *Homogeneity*. In order to reduce operational costs, districts should have homogeneous metering workloads, measured in terms of number of customers and reading time. The second objective function (2) minimizes the workload deviations from the targets.
(3) *Conformity*. In case of preexisting districts, utilities may wish not to abruptly change the current configuration as this could lead to high operational costs. The number of customer-district reassignments allowed to occur is limited in the MCRP model by constraint (4). Section 5.4 shows how the conformity can be determined for a solution.

4. Node and edge partitioning

Solutions techniques for the CDP are usually based on node partitioning. However, in this application the demands, namely reading time and number of customers, lie on the edges. Therefore, in order to use a node partitioning method for the MCRP, we