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The multi-district team orienteering problem

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ABSTRACT

This paper introduces the multi-district team orienteering problem. In this problem, one must schedule a set of mandatory and optional tasks located in several districts, within a planning horizon. The total available time determined by the length of the planning horizon must be distributed among the districts. All mandatory tasks within each district must be performed, while the other tasks can be performed if time allows. A positive profit or score is collected whenever an optional task is performed. Additionally, some incompatibility constraints between tasks are taken into account. The objective is to determine a schedule for a set of tasks to be performed daily within each district, while maximizing the total collected profit. A mixed integer formulation and an adaptive large neighborhood search heuristic are proposed for this problem. The performance of the proposed algorithm is assessed over a large set of randomly generated instances. Computational results confirm the efficiency of the algorithm.

1. Introduction

The problem studied in this work arises from a real situation faced by the Ministry of Transport in the province of Québec in Canada. Periodically, every three or six months, the Ministry must plan its road maintenance activities such as repairs or road marking. The relevant region is divided into districts, to each of which are associated mandatory and optional maintenance activities. All mandatory tasks must of course be performed over the planning horizon, but optional tasks are only executed if time allows. When an optional task i is performed, it generates a profit or score s_i. All districts must be served, and the total available time within the planning horizon must be split among the districts. The amount of time assigned to each district is determined during the planning stage.

Once the total time assigned to each district is determined, the scheduling of activities is carried out by considering incompatibility constraints between tasks and ensuring that all mandatory tasks are performed. Incompatibilities between tasks can be the result of a number of policies. The most common occurs when a road needs repairs in both directions, in which case repairs cannot be carried out in the two directions on the same day because one cannot close both sides of the road simultaneously. Mandatory tasks are related to repairs that are vital to keep a road in a functional state, while optional tasks are those that can wait for the next planning horizon without disabling the road. Usually, no more than 50% of the total repairs are mandatory, while the rest are optional.

In this application, all districts are served in turn by a team of workers based at a depot which remains open until the scheduled tasks are completed. Every day, the team leaves the depot to perform the schedule for that day and then returns to the depot. A working day has a duration of 12 h and a working week contains six working days. A team operates within the same district during a certain number of full weeks.

The problem consists of constructing a schedule of maintenance work over the planning horizon so that all mandatory tasks are performed, and the total score associated with the optional tasks that are performed is maximized. Each daily schedule is viewed as a route in the context of vehicle routing. To the best of our knowledge, this problem has never been addressed in the scientific literature. We call it the *Multi-District Team Orienteering Problem (MDTOP)* because the scheduling problem in each district can be viewed as the *Team Orienteering Problem (TOP)* with additional constraints.

The TOP was introduced by Chao et al. [3]. In the TOP, given a fixed amount of time for each one of the m members of a team, the goal is to determine m routes starting and ending at a specific point through a subset of locations in order to maximize the total score. The TOP is rooted in the *Orienteering Problem (OP)*, also known as the *Selective Traveling Salesman Problem (STSP)* (see e.g., [12,5–7]). Tang and Miller-Hooks [11] have proposed a tabu search

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algorithm for the TOP, whereas Boussier et al. [2] have developed a branch-and-price algorithm for the same problem. Archetti et al. [1] have proposed two variants of a generalized tabu search algorithm and a variable neighborhood search algorithm for the TOP and have shown that each of these algorithms outperforms the heuristics of [11].

A variant of the TOP, called the TOPTW, was later introduced by Vansteenwegen et al. [13]. In the TOPTW each vertex has an associated time window. The authors have proposed an integer linear programming formulation and an iterated local search heuristic for this problem. The same authors [14] have also proposed a path relinking algorithm for the TOP which outperforms the previous heuristics of Tang and Miller-Hooks [11] and of Archetti et al. [1], among others.

In the MDTOP there are several districts, each of which contains a subset of mandatory tasks (this feature is not included in the TOP) and another subset of optional tasks. Additionally, the MDTOP contains incompatibility constraints which are not present in the TOP. In this paper, we propose a mixed integer linear programming formulation, as well as an adaptive large neighborhood search (ALNS) metaheuristic for the MDTOP.

The organization of this work is as follows. Section 2 formally describes the MDTOP and provides a mathematical model for it. Section 3 describes the proposed ALNS metaheuristic for the problem under study. Computational results are presented in Section 4, followed by conclusions in Section 5.

2. Formal problem description and model

Given a region partitioned into a set D of districts and a planning horizon of h weeks, the MDTOP consists of determining the number w_d of weeks allocated to each district $d \in D$ and the schedule of all tasks to be performed on each day of the w_d weeks. The total time allocated to the districts cannot exceed the length of the planning horizon h, which means that $\sum_{d \in D} w_d \le h$. To each district sets of mandatory and optional tasks, called M_d and O_d, respectively, are assigned. Within the time w_d assigned to district d, all mandatory tasks in M_d must be completed, and some optional tasks in O_d can also be performed. All tasks, mandatory and optional, consume some service time if they are performed, and if an optional task $i \in O_d$ is executed, a non-negative score s_i is then collected. The aim of the MDTOP is to perform all mandatory tasks and possibly some optional tasks in such a way that the total collected score is maximized, the total time assigned to each district d does not exceed w_d, and some side constraints are satisfied. Namely, the maximum duration of a working day is 12 h, each week contains six consecutive working days, and w_d is integer. Moreover, there are some incompatible tasks which cannot be carried out during the same day.

2.1. Mathematical model

Let $G_d = (V_d, A_d)$ be a complete graph representing a district $d \in D$, where the vertex set $V_d = \{M_d \cup O_d \cup \{e_d, \overline{e}_d\}\}$ contains all tasks in district d, and two copies e_d and $\overline{e}_d$ of the depot in district d. Let a_j^d be the time needed to perform task j in district d, where $j \in M_d \cup O_d$. The set A_d is the arc set in district d. The travel time t_{ij} from vertex i to vertex j is known for all $(i,j) \in A_d$. Each $i \in O_d$ has an associated profit s_i. Some tasks $j \in M_d \cup O_d$ have an associated set of tasks $C_j^d \subset V_d \backslash \{e_d, \overline{e}_d, j\}$ which cannot be performed on the same day as j.

The sequence of tasks assigned to a team during a working day is a route starting and ending at the depot and whose duration does not exceed a time limit T_{max} equal to 12 h in our application. Since there are $b=6$ working days in a week, the work performed by the same team over a week in the same district can be represented by a set of six routes.

For a better explanation of our model, we define the maximum number of routes per district as $P_d = \min\{hb, |V_d|\}, d \in D$. The decision variables are defined as follows:

$$x_{ij}^{pd} = \begin{cases} 1 & \text{if in route } p \text{ in district } d, \text{ a task performed at vertex } i \\ & \text{is followed by a task performed at vertex } j, i, j \in V_d \\ 0 & \text{otherwise.} \end{cases}$$

$$y_i^{pd} = \begin{cases} 1 & \text{if a task at vertex } i \text{ is part of route } p \text{ in district } d \in D, \ i \in V_d \\ 0 & \text{otherwise.} \end{cases}$$

w_d is the number of working weeks assigned to district d, $d \in D$.
u_i^{pd} represents the position of vertex i in route p in district d, $i \in V_d$.

The problem is then formulated as follows:

$$(MDTOP) \text{ Maximize } \quad z = \sum_{d \in D} \sum_{p \in P_d} \sum_{i \in O_d} s_i y_i^{pd} \tag{1}$$

subject to

$$\sum_{p \in P_d} \sum_{i \in V_d \backslash \{e_d\}} x_{e_d i}^{pd} \le b w_d \quad d \in D \tag{2}$$

$$\sum_{p \in P_d} \sum_{i \in V_d \backslash \{\overline{e}_d\}} x_{i \overline{e}_d}^{pd} \le b w_d \quad d \in D \tag{3}$$

$$\sum_{p \in P_d} y_k^{pd} = 1 \quad d \in D, k \in M_d \tag{4}$$

$$\sum_{p \in P_d} y_k^{pd} \le 1 \quad d \in D, k \in O_d \tag{5}$$

$$\sum_{i \in V_d \backslash \{\overline{e}_d\}} x_{ik}^{pd} = y_k^{pd} \quad d \in D, p \in P_d, k \in M_d \cup O_d \tag{6}$$

$$\sum_{i \in V_d \backslash \{e_d\}} x_{ki}^{pd} = y_k^{pd} \quad d \in D, p \in P_d, k \in M_d \cup O_d \tag{7}$$

$$\sum_{i \in V_d \backslash \{\overline{e}_d\}} \sum_{j \in V_d \backslash \{e_d\}} t_{ij} x_{ij}^{pd} + \sum_{i \in V_d \backslash \{e_d, \overline{e}_d\}} a_i^d y_i^{pd} \le T_{max} \quad d \in D, p \in P_d \tag{8}$$

$$\sum_{d \in D} w_d \le h \tag{9}$$

$$y_i^{pd} \le u_i^{pd} \le (|V_d| - 2) y_i^{pd} \quad d \in D, p \in P_d, i \in V_d \backslash \{e_d, \overline{e}_d\} \tag{10}$$

$$u_i^{pd} - u_j^{pd} + 1 \le (|V_d| - 2)(1 - x_{ij}^{pd}) \quad d \in D, p \in P_d, i, j \in V_d \backslash \{e_d, \overline{e}_d\} \tag{11}$$

$$\sum_{j \in C_i^d} y_j^{pd} \le 1 - y_i^{pd} \quad d \in D, i \in V_d \backslash \{e_d, \overline{e}_d\}, C_i^d \ne \emptyset, p \in P_d \tag{12}$$

$$x_{ij}^{pd}, y_i^{pd} \in \{0, 1\} \quad i, j \in V, p \in P_d \tag{13}$$

$$u_i^{pd} \in \mathbb{Z} \quad i \in V, p \in P_d \tag{14}$$

$$w_d \in \mathbb{Z}^+ \quad d \in D. \tag{15}$$

Objective (1) computes the total collected score of the selected optional tasks. Constraints (2) and (3) state that each route starts and ends at the depot and the number of routes does not exceed the maximum number of days assigned to each district d. Constraints (4) guarantee that each mandatory task is performed. Constraints (5) ensure that an optional task is selected at most once. Constraints (6) and (7), combined with (4) and (5), enforce the flow conservation conditions at the vertices. Constraints (8) mean that the maximum available time T_{max} is not exceeded on any route. Constraint (9) guarantees that all tasks are completed