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Optimization of robust area traffic control with equilibrium flow under demand uncertainty

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ABSTRACT

For area traffic control road network under realization of uncertain travel demand, a robust signal setting is investigated in this paper. Due to certain hierarchy in a decision-making order, a min–max bilevel program is proposed. A new solution method is presented to determine a Nash–Stackelberg solution where a proposed signal setting is found for area traffic control under demand uncertainty. In order to investigate the robustness of the proposed signal settings, numerical computations are performed for various initial data sets in a medium-sized example road network. Good computational results indicated that the proposed signal settings can successfully reduce a worst-case travel cost substantially while incurring a relatively slight loss of optimality with respect to the optimal deterministic solutions for nominal travel demands. Particularly, our computation results showed that the proposed signal settings become even attractive as demand growth increases under a worst-case realization taken by uncertain travel demands.

1. Introduction

In urban areas most travel delays incurred by road users are occurring at signal-controlled junctions at which traffic movements are interrupted regularly by alternating traffic lights. In urban area traffic control road network, planning to both cope with continuously growing travel demand and alleviate increasing traffic congestion at signalized junctions becomes one of the most challenging and important issues facing decision makers at various levels of management. Optimization of signal settings with certain travel demand in a road network has attracted many researchers over past decades [1–8]. For example, TRANSYT [9] is recognized as one of the most useful tools in studying optimization of signal timings. As indicated in [10], traffic flows and travel times are strongly influenced by operations of signals. Ways of using mathematical programming to solve a constrained optimization problem of signal timings and equilibrium flow have been well researched [11–14]. Allsop and Charlesworth [11] were first ones who proposed a mutually consistent calculation, which was also known as an iterative optimization assignment (IOA for short). IOA for area traffic control optimization with equilibrium flow is to update signal settings for current flow and alternately solve a traffic assignment for the resulting signal settings until an intuitively expected convergence is achieved. The resulting mutually consistent signal timing and equilibrium flow will, however, in general be a non-optimal solution. On the other hand, a bilevel programming technique traditionally has been considered a more appropriate tool in tackling signal setting problem (see [15,16]) with certain travel demand. Yang and Yagar [15], for instance, proposed a sensitivity analysis based algorithm to solve a nonlinear inequality constrained problem for green splits in a signal-controlled road network. According to [7], a traffic control manager with the leader at the upper level is supposed to have all information about road travelers' objective function and constraints, while at the lower level road travelers with the followers know nothing about the leader but the strategy announced by him. Until the leader announces his own optimal strategy, the followers cannot solve their optimizing problem responding to the leader's strategy. A Stackelberg solution is an optimal strategy for the leader when followers (travelers) react by playing optimally. However, evaluating network performance simply using nominal data for travel demand will lead to erroneous selection of signal settings and overestimates overall system performance particularly when traffic condition is fluctuated over time. The work in [7,8,15,16] restricted signal settings to a certain level of travel demand and no evaluation was given for effect on traffic conditions of travel demand uncertainty of future growth.

The problem of determining computationally tractable signal settings that are feasible for any realization of uncertain demand attracts growing interest in the fields of operations research and transportation science. Considering a network design with uncertain input data, there are two mostly common used approaches in

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the literature: stochastic programming and robust optimization [17]. From the prospective of stochastic programming, given a known priori distribution of probabilities of uncertain data, there are a variety of approaches applied to road network design with uncertain demand such as chance-constrained programming and reliability-based approach (see [18–21]). A growing interest in robust optimization approach [22–31] has attracted various applications in the field of decision science. A scenario-based robust solution in [22,26–28] is presented for large-scale network design in which a stochastic linear programming approach is employed. Furthermore, assuming uncertain data bounded within some certain set, there are plenty research work extending a robust optimization in [23–25] to road network design with uncertain demand such as [29–31]. However, considering area traffic control in the presence of travel demand uncertainty, at the best of author's knowledge, there is very limited research work with exception of [32,33]. In [32] a system optimum based signal control accounting for traffic dynamics was considered. A robust dynamic system optimal model was built to quantify network performance over the range of possible scenarios of uncertainty. However, the behavior of road users' route choice is not fully taken into account when the signal control is optimal with respect to uncertainty of traffic dynamics. On the other hand, a risk-averse user equilibrium model with certain demand in [33] is presented for route choice in a signal-controlled network where travel time and accident risk rate are evaluated by simple functions. No explicit mathematical expression is allowed for signal co-ordination in [33] and platoon dispersion on adjacent streets are not taken into account.

In order to effectively enhance the capacity of area traffic road network and mitigate delay incurred to all travelers in the presence of travel demand surge and disruptive events, in this paper a robust area traffic control (RAC) is proposed for equilibrium flow. A robust signal setting is proposed to minimize journey times for all road travelers under a realization of demand uncertainty. The underlying behavior of travelers' route choice following Wardrop's first principle is taken into account. Since the precedence in decision-making order for signal-controlled road network exists, a Stackelberg solution for a two-level area traffic control with equilibrium flow is considered. At the upper level the manager with the leader determines optimal signal settings which can be regarded as parameters for the lower level problem, whilst at the lower level a road traveler with the follower minimizes his journey time with respect to route choice under these parameters. In this paper, a worst-case analysis is investigated under which a realization taken by future demand that is most unfavorable is considered. A min–max bilevel programming approach is proposed to formulate a RAC problem with equilibrium flow in the presence of uncertain travel demand. It is assumed that total travelers delay is minimized over the set of signal settings that are feasible for a worst-case realization taken by future travel demand. The performance index (PI) evaluated at the upper level taking reaction of road travelers into account, maximized with respect to a future demand growth factor, is minimized with respect to signal setting variables. For a general bilevel programming program, decision variables at the upper level are optimized subject to the solutions of lower level problem. As it noted by [34–37] that in most cases a solution of lower level problem is not mathematically explicit and cannot be fully expressed as a closed form, a bilevel program turns out to be a non-convex problem. There are computationally tractable algorithms [38–41] following the first-order sensitivity analysis of equilibrium constraint [42] for a bilevel network design problem. Because of the non-linearity of equilibrium constraint with respect to decision variables, solution algorithms mentioned above [15,16,41,43] can simply solve a non-convex constrained bilevel problem only locally. As noted from the literature [44–46], the equilibrium flow is generally not differentiable at some point such that the first-order approximation for equilibrium flow may fail at these points. Therefore, it would be preventive from direct use of the results of sensitivity analysis in [42] for equilibrium flows. In this paper, we propose a new bundle-type gradient approach using the generalized gradients of PI to tackle this problem following recent work in [47–49] where the first-order directional derivatives for general equilibrium flow exist almost everywhere.

The contributions made from this paper are summarized as follows. Firstly, a robust signal setting is presented. A min–max bilevel programming approach is proposed to formulate a robust signal setting problem under a realization taken by travel demand growth that is most unfavorable. A Nash–Stackelberg solution can be found such that the value of PI, maximized with respect to travel demand growth factor, is minimized with respect to a proposed signal setting subject to equilibrium flow. The worst-case PI value serves as an upper bound estimate for the RAC problem with equilibrium flow. The equilibrium flow following Wardrop's first principle for a realization taken by uncertain travel demand which is most unfavorable is determined by a parameterized variational inequality. Secondly, a computationally tractable solution procedure is proposed for a RAC problem. A bundle-type gradient approach is presented in which the generalized gradients for the RAC problem are derived. Based on recent work in sensitivity analysis, the first-order directional derivatives are derived for equilibrium flow with respect to robust signal settings and travel demand growth factor. Thirdly, numerical computations are performed using a medium-sized road network where signal-controlled junctions are particularly considered. As it mentioned above, a Nash–Stackelberg solution for area traffic control with equilibrium flow under demand uncertainty can be regarded as a worst-case scenario in which the feasibility of solution is pursued at expense of optimality. In order to evaluate the effectiveness and robustness of a proposed signal setting under a worst-case realization of demand growth factor, two robustness measures for the RAC problem with equilibrium flow are therefore presented. Computational results obtained indicate that a proposed signal setting for a RAC problem with equilibrium flow can successfully reduce the worst-case PI value substantially while incurring a relatively slight loss of optimality with respect to the optimal deterministic signal settings for nominal travel demands. Particularly, our computation results showed that the proposed signal settings become even attractive as travel demand growth factor increases under a worst-case realization taken by uncertain travel demands.

The rest of the paper is organized as follows. Section 2 introduces a RAC problem formulation with equilibrium flow. A min–max bilevel programming approach is proposed for a robust signal setting problem in the presence of uncertain travel demand. A parametric equilibrium traffic assignment is formulated as a variational inequality. The first-order directional derivatives of equilibrium flow are derived. In Section 3, a bundle-type gradient method with projection is presented to improve the solution found for the RAC problem with equilibrium flow. Numerical computations are performed in Section 4 using a medium-sized road network where signal-controlled junctions are particularly considered. Conclusions for this paper and extensions of the proposed approach to topics of interest are briefly summarized in Section 5.

2. A robust area traffic control problem under demand uncertainty

A min–max bilevel programming is proposed to formulate a robust area traffic control (RAC) with equilibrium flow in the presence of uncertain travel demand. A RAC problem with