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Effective replenishment policies for the multi-item dynamic lot-sizing problem with storage capacities

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ABSTRACT

We address the dynamic lot-sizing problem considering multiple items and storage capacity. Despite we can easily characterize a subset of optimal solutions just extending the properties of the single-item case, these results are not helpful to design an efficient algorithm. Accordingly, heuristics are appropriate approaches to obtain near-optimal solutions for this NP-hard problem. Thus, we propose a heuristic procedure based on the smoothing technique, which is tested on a large set of randomly generated instances. The computational results show that the method is able to build policies that are both easily implemented and very effective, since they are on average 5% above the best solution reported by CPLEX. Moreover, an additional computational experiment is carried out to show that the performance of this new heuristic is on average better and more robust than other methods previously proposed for this problem.

1. Introduction

One of the most critical issue that should be tackled by a firm is the determination of the procurement policy of the different goods (spare parts, raw material, components or finished items) involved in the supply chain. This problem becomes more complex whether the demand for each of the N items varies with time through a finite planning horizon with T periods. Accordingly, problems in this category are usually referred to as multi-item dynamic lot-sizing model. Generally speaking, three main aspects (criteria) can influence on the replenishment decisions: cost minimizing, strategic (e.g., demand fulfillment) and limiting factors (e.g., storage constraints). The cost minimizing is a primordial goal in most economic activities since it usually entails profit increasing and hence allows the firm to occupy a better position within a global competitive environment. However, this goal can be in conflict with the others two criteria. On the one hand, the unplanned shortages can yield both additional costs due to backlogging situations or interruptions of the production process and a loss of customer's confidence. Furthermore, the decision maker should keep in mind the storage limitations to ensure the procurement quantities can be stored in the available space and hence to guarantee the feasibility of the policy. The real-world applications where this heuristic can be helpful are (Minner [11]): JRP (Joint Replenishment Problems), Capacitated Lot-sizing and Scheduling Problems, and Warehouse Scheduling Problems.

Wagner and Whitin [14] were the first to study the dynamic version of the Economic Order Quantity (EOQ) model and since then a significant number of papers has been published considering diverse extensions. These variants have been successfully implemented in both dependent (vertical) and independent (horizontal) demand systems (see, e.g., Brahimi et al.[3] and Robinson et al. [12]). One of these extensions is credited to Love [10], who developed a $O(T^3)$ procedure to optimally solve the single-item single-stage dynamic lot size problem with limited capacity at the warehouse and general concave cost structure for the inventory, ordering and backlogging operations. More recently, Gutiérrez et al. [8,9] introduced a characterization to the optimality, which led to devise an algorithm with the same theoretical complexity as that in Love, but with running times almost 30% faster than the former. Furthermore, the algorithm runs in $O(T)$ expected time when the upper bound of the demand at each period coincides with its storage capacity.

Contributions to the case with multiple items are quite sparse. For instance, Dixon and Poh [4] proposed a smoothing approach, which consists of determining independently the replenishment policy for each product using any of the efficient algorithms proposed by Argawal and Park [1], Federgruen and Tzur [5] or Wagelmans et al. [13]. In a second step, if the inventory level at the end of period t exceeds the storage capacity then, this infeasibility is corrected either by postponing from period t to t+1 the corresponding procurement quantity (PUSH operation) or by advancing the next replenishment for an item (PULL operation). Moreover, Günther [6,7] devised a constructive approach, which consists of determining first lot-for-lot policies and then successively increasing the replenishment quantities for each item and period using an economic criterion as a discriminant. Inspired by the well-known savings algorithm for the vehicle routing problem, Axsäter [2] introduced a heuristic approach for the

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dynamic lot-sizing problem, which consists of reducing the ordering costs by combining consecutive replenishments in one batch whenever this operation represents a saving. These three approaches were implemented and compared in a comprehensive article by Minner [11], and the results show that both the constructive and the smoothing techniques have an antagonistic behavior of the saving approach. In other words, this last method is more robust than the other two heuristics when demand variability increases and shows smaller increases of deviations for variations of costs and capacity parameters.

We address in this paper the problem of determining the near-optimal ordering schedules for a set of items, which share a common warehouse within an independent (horizontal) demand system. Moreover, a characterization of a family of optimal solutions is derived as a straightforward extension of that for the single-item version in Gutiérrez et al. [8]. This characterization allows us to develop an improved version of the PUSH operation proposed by Dixon and Poh [4], where infeasibility is not only fixed postponing the demand of only one period but the size of the deferred batch for an item corresponds to either the sum of demands of consecutive periods or a quantity enough to saturate the storage capacity. Furthermore, this approach permits to devise an $O(N^2\,T^2 \log T)$ algorithm which provides reasonably good near-optimal solutions that are on average 5% above the best solution achieved by CPLEX.

The remainder of the paper is organized as follows. In Section 2 we formulate the problem. The key results that allows to improve the approach proposed by Dixon and Poh [4] and the heuristic method are described in detail in Section 3. Moreover, the new approach is illustrated with a simple example in Section 4. In Section 5, we explain how the computational experiment has been performed and computational results for several sets of randomly generated instances are reported. Furthermore, and additional computational experiment is carried out to show the good performance of the new heuristic in comparison with other methods previously proposed. Finally, in Section 6 we provide some conclusions and final remarks.

2. Problem statement

We assume a set of N items independently demanded and a planning horizon with T periods. For each item n and period t we define the following parameters. Let $d_{n,t}$, $f_{n,t}$, $p_{n,t}$ and $h_{n,t}$ be respectively the demand, the fixed setup cost, the production cost and the carrying cost for item n in period t. Moreover, we denote by $D_{n,t}$ the accumulated demand of item n from periods t to T, that is $D_{n,t}=\sum_{k=t}^{T} d_{n,k}$. Additionally, let S_t be the total dynamic inventory capacity at the warehouse in period t and let w_n be the unit capacity (volume) of item n. On the other hand, for each item n and period t, we define the following variables, the order quantity $x_{n,t}$ replenished at the beginning of the corresponding period, the binary variable $y_{n,t}$ which is equal to 1 if item n is ordered in period t and 0 otherwise, and the inventory quantity $I_{n,t}$ of item n at the end of period t. Moreover, we assume that shortages are not permitted and leadtimes are negligible. Thus, we can state the following MIP problem:

$$\min \quad \sum_{n=1}^{N}\sum_{t=1}^{T} f_{n,t}y_{n,t}+p_{n,t}x_{n,t}+h_{n,t}I_{n,t} \tag{1}$$

$$\text{s.t.}: \quad I_{n,0}=I_{n,T}=0, \quad n=1,\ldots,N \tag{2}$$

$$\sum_{n=1}^{N} w_n(I_{n,t-1}+x_{n,t}) \le S_t, \quad t=1,\ldots,T \tag{3}$$

$$I_{n,t-1}-I_{n,t}+x_{n,t}=d_{n,t}, \quad n=1,\ldots,N;\ t=1,\ldots,T \tag{4}$$

$$x_{n,t}\le y_{n,t}D_{n,t}, \quad n=1,\ldots,N;\ t=1,\ldots,T \tag{5}$$

$$x_{n,t}, I_{n,t}\in\mathbb{N}_0=\mathbb{N}\cup 0;\ y_{n,t}\in\{0,1\}, \quad n=1,\ldots,N;\ t=1,\ldots,T \tag{6}$$

The terms in (1) represent, respectively, the total setup cost, the total ordering cost and the total holding cost. The set of constraints in (2) forces both the initial and final inventory level of the planning horizon for each item to be zero. However, these constraints can be dropped off without loss of generality since the case of positive initial and/or final inventory can be adapted to the formulation above just allocating initial inventories to demands of the first periods and/or adding a required final inventory to the demand of the last period. The second set of constraints (3) ensures that the inventory level at the beginning of each period does not exceed the warehouse capacity. The constraints in (4) are the well-known material balance equations and those in (5) state the relationship between the order quantity and its binary variable for each item and period. Finally, constraints in (6) define the character of each variable.

3. The heuristic method

Among the different approaches proposed to obtain near-optimal solutions to the problem above (see Minner [11]), we focus our attention on that presented by Dixon and Poh [4]. These authors devised a *smoothing* approach based on solving first a relaxed version of the problem dropping off the constraint set in (3). Thus, independent plans are obtained by solving N single-product, dynamic uncapacitated lot-sizing problems. If infeasibility occurs at the end of period t, the excess warehouse capacity requirements can be reduced either by postponing an item replenishment batch from period t to $t+1$ (PUSH operation) or by advancing the next future replenishment (PULL operation). However, Dixon and Poh claimed that on average the PUSH operation is preferred to the PULL strategy since the impact of the move on total costs is predictable. Moreover, we consider that this approach has not been appropriately exploited since it just moves the sum of demands of consecutive periods for several items instead of filling the warehouse capacity. Accordingly, we confine ourselves to analyze the PUSH operation but extending the strategy to consider both postponements of orders for a subset of items to nonconsecutive periods (i.e., from period t to period $t+k$, $k\in\{1,\ldots,T-t\}$, instead of $t+1$ only) and replenishment quantities different from a sum of demands of consecutive periods for an item. This last modification is supported by the results in Gutiérrez et al. [8]. Specifically, the natural extension of Theorem 1 in Gutiérrez et al. suggests that in each of the production periods t (i.e., those periods for which $x_{n,t}\neq 0$ for some item n), the sum of both inventory and production quantities for all items matches either the sum of demands of successive periods, or the maximum storage capacity for that period. This property is just a generalization of Theorem 1 in [8] to the multi-item case, which can be formally stated in the following theorem.

Theorem 3.1. *If t is a production period ($x_{n,t}\neq 0$ for some item n), then either $I_{n,t-1}+x_{n,t}=D_{n,t}-D_{n,k_{n,t}}$ for some period $k_{n,t}>t$ or $\sum_{n=1}^{N}(I_{n,t-1}+x_{n,t})=S_t$.*

Proof. For a contradiction, let us consider an optimal solution in which there is an item n with non-zero production in period t ($x_{n,t}\neq 0$), for which $I_{n,t-1}+x_{n,t}>D_{n,t}-D_{n,k_{n,t}}$ and $I_{n,t-1}+x_{n,t}<D_{n,t}-D_{n,k_{n,t}+1}$, where $k_{n,t}\in\{t+1,\ldots,T-1\}$. Besides, let us admit that $\sum_{n=1}^{N}(I_{n,t-1}+x_{n,t})<S_t$. From this point, we can easily proceed in the same manner than in the proof of Theorem 1 in Gutérrez et al. [8] to ensure that either the current solution is not optimal or we can construct an optimal plan satisfying the condition of the statement.