Computers & Operations Research 40 (2013) 2906–2929

Contents lists available at SciVerse ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Several flow shop scheduling problems with truncated position-based learning effect

Xiao-Yuan Wang [a,b,e,*], Zhili Zhou [a], Xi Zhang [a], Ping Ji [c,d], Ji-Bo Wang [b,c,e,*]

[a] School of Management, Xi'an Jiaotong University, Xi'an 710049, China
[b] School of Science, Shenyang Aerospace University, Shenyang 110136, China
[c] Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, China
[d] College of Transportation Management, Dalian Maritime University, Dalian 116026, China
[e] State Key Laboratory for Manufacturing Systems Engineering, Xi'an Jiaotong University, Xi'an 710053, China

ARTICLE INFO

Available online 11 July 2013

Keywords:
Scheduling
Learning effect
Regular performance criteria
Worst-case analysis
Flow shop
Heuristic algorithm

ABSTRACT

The concept of truncated position-based learning process plays a key role in production environments. However, it is relatively unexplored in the flow shop setting. In this paper, we consider the flow shop scheduling with truncated position-based learning effect, i.e., the actual processing time of a job is a function of its position and a control parameter in a processing permutation. The objective is to minimize one of the six regular performance criteria, namely, the total completion time, the makespan, the total weighted completion time, the discounted total weighted completion time, the sum of the quadratic job completion times, and the maximum lateness. We present heuristic algorithms and analyze the worst-case bound of these heuristic algorithms. We also provide the computational results to evaluate the performance of the heuristics.

1. Introduction

Scheduling problems have received considerable attention for many years. In traditional scheduling problems, most research assumes that job processing times are known fixed values in advance, however, there are many realistic settings, because firms and employees perform a task over and over, they learn how to perform more efficiently. The production facility (a machine, a worker) improves continuously over time. As a result, the production time of a given product is shorter if it is scheduled later, i.e., unit costs decline as firms produce more of a product and gain knowledge or experience. This phenomenon is known as the "learning effect" in the literature. Extensive surveys of different scheduling models and problems involving jobs with learning effects can be found in Biskup [2] and Janiak and Rudek [12]. More recent papers which have considered scheduling jobs with learning effects include Wu et al. [38], Koulamas and Kyparisis [16], Eren and Güner [5], Wang et al. [30], Xu et al. [42], Mosheiov and Sarig [22], Janiak et al. [14], Lee and Wu [18], Janiak and Rudek [13], Cheng et al. [3], Rudek [26], Mor and Mosheiov [19], Mosheiov [20], Wang and Wang [31,32], Mosheiov and Oron [21], Wu and Lee [35–37], Gerstl and Mosheiov [8], Wu et al. [39], Lee [17], Wu et al. [40], Yin et al. [43], and Wu et al. [41].

However, subject to an uncontrolled learning effect, the actual processing time of a job will plummet to zero precipitously as the number of jobs increases in the job position-based learning model proposed by Biskup [1]. Motivated by this observation, Wu et al. [40] proposed a truncated position-based learning model where the actual processing time of a job is a function of its position and a control parameter, i.e., the actual processing time of J_j is defined as $p_{jr} = p_j \max\{r^a, \rho\}$, where p_j is a normal processing time of job J_j, $a \le 0$ is the learning index and ρ is a truncation parameter with $0 < \rho < 1$. The use of the truncated function can be justified on the grounds that learning, like other human activities, is limited. They showed that even with the introduction of the proposed model to job processing times, several single machine problems and two-machine flows hop problems remain polynomially solvable. They also analyzed the worst-case error bounds for the problems to minimize the total weighted completion time, discounted total weighted completion time and maximum lateness. To the best of our knowledge, the concept of truncated position-based learning process is relatively unexplored in flow shop environment. Wu et al. [40] are the only authors to study the two-machine flow shop problem under the assumption of truncated position-based learning functions. In this paper we continue the work in Wu et al. [40] considering flow shop scheduling problems with truncated position-based learning effects. The objective is to minimize one of the six regular performance criteria, namely, the total completion

* Corresponding author at: School of Science, Shenyang Aerospace University, Shenyang 110136, China. Tel.: +86 24 89723548.
E-mail addresses: wxy5099@126.com (X.-Y. Wang), wangjibo75@163.com (J.-B. Wang).

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http://dx.doi.org/10.1016/j.cor.2013.07.001

time, the makespan, the total weighted completion time, the discounted total weighted completion time, the sum of the quadratic job completion times, and the maximum lateness. We present heuristic algorithms with worst-case bound for each criterion by utilizing the optimal permutations for the corresponding single machine problems.

The remaining sections are organized as follows. In Section 2, we give some general notations and assumptions. In Sections 3–8, we propose heuristic algorithms with a worst-case bound for the total completion time minimization, the makespan minimization, the total weighted completion time minimization, the discounted total weighted completion time minimization, the sum of the quadratic job completion times minimization and the maximum lateness, respectively. In Section 9, we give some well-known heuristics. In Section 10, we present the computational experiments. Conclusions and remarks are given in Section 11.

2. Notations and assumptions

The details of the flow shop scheduling with truncated position-based learning effect are described as follows: A set of n jobs $J_1, J_2, \ldots, J_n$ are to be processed on m continuously available flow shop machines $M_1, M_2, \ldots, M_m$. Each job J_j consists of chain operations $(O_{1j}, O_{2j}, \ldots, O_{mj})$. Operation O_{ij} has to be processed on machine M_i, $i = 1, 2, \ldots, m$. The starting time of operation O_{ij} must be the larger one of the completion times of $O_{i-1,j}$ and $O_{i,j-1}$ and all machines process the jobs in the same permutation schedule. As in Wu et al. [40], we consider the model of actual job processing time p_{ijr} characterized by truncated position based learning function, i.e.,

$$p_{ijr} = p_{ij} \max\{r^a, \rho\}, \quad i = 1, 2, \ldots, m, \quad r, j = 1, 2, \ldots, n, \tag{2.1}$$

where p_{ij} denotes the (normal) processing time of operation O_{ij}, $a \le 0$ is a learning ratio and ρ is a truncation parameter with $0 \le \rho \le 1$.

Let $C_{ij} = C_{ij}(S)$ be the completion time of operation O_{ij}, $C_j = C_{mj}$ be the completion time of job J_j, $S = (J_{[1]}, J_{[2]}, \ldots, J_{[n]})$ represent a permutation of $(1, 2, \ldots, n)$, where $[j]$ denotes the job that occupies the jth position in S. The aim of this paper is to seek a sequence for minimizing $\sum_{j=1}^{n} C_j$ (the total completion time), $C_{\max} = \max\{C_j | j = 1, 2, \ldots, n\}$ (the makespan), $\sum_{j=1}^{n} w_j C_j$ (the total weighted completion time, where $w_j > 0$ is a weight associated with job J_j), $\sum_{j=1}^{n} w_j (1 - e^{-\gamma C_j})$ (the discounted total weighted completion time, where $\gamma \in (0, 1)$ is the discount factor (see [24, Section 3.1])), $\sum_{j=1}^{n} C_j^2$ (the sum of the quadratic job completion times [29]) and $L_{\max} = \max\{L_j | j = 1, 2, \ldots, n\}$ (the maximum lateness), where d_j denote the due date of job J_j and $L_j = C_j - d_j$. According to the standard three-field notation for scheduling problems [10], the problem will be denoted as $Fm|prmu, p_{ijr} = p_{ij} \max\{r^a, \rho\}|F$, $F \in \{\sum C_j, C_{\max}, \sum w_j C_j, \sum w_j (1 - e^{-\gamma C_j}), \sum_{j=1}^{n} C_j^2, L_{\max}\}$.

3. Worst-case behavior for the total completion time minimization problem

3.1. Case for $m = 2$

Lemma 3.1. *For the $1|p_{jr} = p_j \max\{r^a, \rho\}|\sum C_j$ problem, an optimal schedule is obtained by sequencing jobs in a non-decreasing order of p_j (i.e., the smallest processing time (SPT) first rule) [40].*

For ease of exposition, we denote p_{1j} by a_j and p_{2j} by b_j. It is well known that the $F2|p_{ijr} = p_{ij} \max\{r^a, \rho\}|\sum C_j$ problem is NP-complete [7]. Hoogeveen and Kawaguchi [10] provided an SPT heuristic for the $F2|prmu|\sum C_j$ problem, i.e., in order of non-decreasing $T_j = a_j + b_j$. From Hoogeveen and Kawaguchi [11] and Lemmas 3.1–3.3, we can use the SPT (in order of non-decreasing $T_j = a_j + b_j$) rule as an approximate algorithm for $F2|prmu, p_{ijr} = p_{ij} \max\{r^a, \rho\}|\sum C_j$ problem. As in Gonzalez and Sahni [9], in examining "worst" schedules, we restrict ourselves to busy schedules, i.e., a schedule in which at all times from start to finish at least one machine is processing an operation. Without loss of generality we assume that $T_1 \le T_2 \le \cdots \le T_n$.

Lemma 3.2. *Let S^* be an optimal schedule. Then*

$$2\sum_{j=1}^{n} C_j(S^*) \ge \sum_{j=1}^{n} (n-j+1)(a_j + b_j) j^a + \sum_{j=1}^{n} b_j n^a + n \min_{1 \le j \le n}\{a_j\}. \tag{3.1}$$

Proof. Consider any schedule $S = (J_{[1]}, J_{[2]}, \ldots, J_{[n]})$, from $C_{2[j]} = \max\{C_{2[j-1]}, C_{1[j]}\} + b_{[j]} \max\{j^a, \rho\} \ge C_{2[j-1]} + b_{[j]} \max\{j^a, \rho\}$, we have $C_{[j]} \ge a_{[1]} + \sum_{k=1}^{j} b_{[k]} \max\{k^a, \rho\}$, for $j = 1, 2, \ldots, n$; hence

$$\sum_{j=1}^{n} C_j \ge n a_{[1]} + \sum_{j=1}^{n} \sum_{k=1}^{j} b_{[k]} \max\{k^a, \rho\} = n a_{[1]} + \sum_{j=1}^{n} (n-j+1) b_{[j]} \max\{j^a, \rho\}. \tag{3.2}$$

In addition, from $C_{2[j]} = \max\{C_{2[j-1]}, C_{1[j]}\} + b_{[j]} \max\{j^a, \rho\} \ge C_{1[j]} + b_{[j]} \max\{j^a, \rho\}$, we have $C_{[j]} \ge \sum_{k=1}^{j} a_{[k]} \max\{k^a, \rho\} + b_{[j]} \max\{j^a, \rho\}$, for $j = 1, 2, \ldots, n$; hence

$$\sum_{j=1}^{n} C_j \ge \sum_{j=1}^{n} \sum_{k=1}^{j} a_{[k]} \max\{k^a, \rho\} + \sum_{j=1}^{n} b_{[j]} \max\{j^a, \rho\}$$
$$= \sum_{j=1}^{n} (n-j+1) a_{[j]} \max\{j^a, \rho\} + \sum_{j=1}^{n} b_{[j]} \max\{j^a, \rho\}. \tag{3.3}$$

From Eqs. (3.2) and (3.3), we have

$$2\sum_{j=1}^{n} C_j(S) \ge \sum_{j=1}^{n} (n-j+1)(a_{[j]} + b_{[j]}) \max\{j^a, \rho\} + \sum_{j=1}^{n} b_{[j]} \max\{j^a, \rho\} + n a_{[1]}$$
$$\ge \sum_{j=1}^{n} (n-j+1)(a_j + b_j) \max\{j^a, \rho\} + \sum_{j=1}^{n} b_j \max\{n^a, \rho\} + n \min_{1 \le j \le n}\{a_j\},$$

as the term $\sum_{j=1}^{n} (n-j+1)(a_{[j]} + b_{[j]}) \max\{j^a, \rho\}$ is minimized by the SPT (in order of non-decreasing $T_j = a_j + b_j$) rule (Lemma 3.1). The lemma follows immediately. □

Let $\delta = \min\{a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n\} \max\{n^a, \rho\}$ and $\beta = \max\{a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n\}$.

Lemma 3.3. *We have that*

$$|a_{k+1} \max\{(k+1)^a, \rho\} - b_k \max\{k^a, \rho\}| \le (\beta - \delta)(a_{k+1} \max\{(k+1)^a, \rho\} + b_k \max\{k^a, \rho\})/(\beta + \delta)$$

for $k = 1, 2, \ldots, n-1$.

Proof. Case (1): Let $|a_{k+1} \max\{(k+1)^a, \rho\} - b_k \max\{k^a, \rho\}| = a_{k+1} \max\{(k+1)^a, \rho\} - b_k \max\{k^a, \rho\}$ for $k = 1, 2, \ldots, n-1$. Suppose to the contrary that $a_{k+1} \max\{(k+1)^a, \rho\} - b_k \max\{k^a, \rho\} > (\beta - \delta)(a_{k+1} \max\{(k+1)^a, \rho\} + b_k \max\{k^a, \rho\})/(\beta + \delta)$ for some k. This inequality can be rewritten as $\beta b_k \max\{k^a, \rho\} < \delta a_{k+1} \max\{(k+1)^a, \rho\}$. Since, $\delta a_{k+1} \max\{(k+1)^a, \rho\} < \delta \beta$, we obtain the contradiction that $b_k \max\{k^a, \rho\} < \delta$.

Case (2): Let $|a_{k+1} \max\{(k+1)^a, \rho\} - b_k \max\{k^a, \rho\}| = b_k \max\{k^a, \rho\} - a_{k+1} \max\{(k+1)^a, \rho\}$ for $k = 1, 2, \ldots, n-1$. The result can be obtained similarly. □

Theorem 3.1. *Let S^* be an optimal schedule and S be an SPT schedule for the $F2|prmu, p_{ijr} = p_{ij} \max\{r^a, \rho\}|\sum C_j$ problem. Then $\sum_{j=1}^{n} C_j(S) / \sum_{j=1}^{n} C_j(S^*) \le 2\beta/(\delta + \beta)$, and this bound is tight.*