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A parallel ordering problem in facilities layout

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ABSTRACT

In this paper, we study a problem that occurs in the row layout of facilities. Among n facilities, suppose that there are t facilities with some characteristic in common so that they should be arranged along one row, leaving the remainder $(n-t)$ facilities to be arranged on a parallel row. The objective is to order the facilities in the two rows such that some cost function is minimized. This problem is called the parallel row ordering problem (PROP). The PROP is a generalization of the single row facility layout problem (SRFLP). Here, a mixed integer programming (MIP) formulation of the PROP is presented which extends a MIP formulation of the SRFLP. We show that a PROP with n facilities may be solved faster than a SRFLP with n facilities. Theoretical and experimental comparisons of the SRFLP and the PROP formulations are presented.

1. Introduction

In facilities layout, a pattern of layout commonly used is the single-row layout by which rectangular facilities are linearly arranged along one row. Given a set $N=\{1,..,n\}$ of facilities, the length ℓ_i of each facility $i\in N$ and the average daily traffic c_{ij} between facilities i and j, $(i,\ j\in N, i<j)$, the single-row facility layout problem (SRFLP) seeks a permutation π of the n facilities, which minimizes a cost function given by a weighted sum of inter-facility distances. Let Π_n be the set of all permutations of N and let d_{ij}^{π} be the center-to-center distance between facilities i and j with respect to a permutation π. Then, the SRFLP was formulated in [1] as follows:

$$\min_{\pi\in\Pi_n} \sum_{i,j\in N; i<j} c_{ij} d_{ij}^{\pi} \tag{1}$$

where

$$d_{ij}^{\pi}=\ell_i/2+\left(\sum_{k\in M_{ij}^{\pi}}\ell_k\right)+\ell_j/2$$

and M_{ij}^{π} is the set of indices of the facilities placed between facilities i and j in the permutation π.

In this paper, we are concerned with an extension of the SRFLP that considers arrangements of the n facilities along more than one row. Let $\{N_i\}_{i=1,\dots k}$ be a partition of N such that $\cup_{i=1}^{k}N_i=N$ and $N_i\cap N_j=\emptyset (1\le i<j\le k)$. Let $R=\{1,\dots,k\}$ be a set of k rows, all parallel to the x-axis. We are given a one-to-one assignment of the set $\{N_i\}_{i=1,\dots k}$ to the set R, so that the facilities pertaining to the subset N_i (for some i, $1\le i\le k$) should be arranged along row r (for some r, $1\le r\le k$). The arrangements in every row should start from a common point. No space is allowed between two adjacent facilities. The facilities have their lengths placed on a row along the x-axis direction. The *distance* between two facilities is assumed to be the rectilinear distance between their centers. As the ordering of facilities occurs in one dimension (x), the other components of the distance do not change between any two facilities. Thus, only the x-distances between facilities will have to be determined.

The objective is to order facilities in the k rows in order to minimize a cost function of the x-distances between facilities. This problem is called the k-Parallel Row Ordering Problem (k-PROP), ($k\ge 2$), although in the case $k=2$ we may simply call it the PROP.

An application of k-PROP is the arrangement of facilities along two or more parallel straight lines on a floor plan (Fig. 1). Another application occurs in the construction of a multi-floor building. Fig. 2 depicts a multi-floor building where the orderings occur along the x-axis.

1.1. Mathematical formulation of the PROP

In this paper, we shall concentrate on the PROP (the case of the problem with $k=2$). Suppose there are t facilities $\{1,\dots,t\}$ that possess a certain characteristic, which makes them to be restricted to the same row (say, Row 1), while the other $(n-t)$ facilities $\{t+1,\dots,n\}$ are restricted to a parallel row (say, Row 2). The subset of facilities placed on a row does not change (just their ordering changes). Therefore, the PROP seeks an ordering π^1 of the t facilities at Row 1 and an ordering π^2 of the $(n-t)$ facilities at Row 2. To formulate the PROP, we define Π_t^1 to be the set of all permutations π^1 of $N_1=\{1,\dots,t\}$; and Π_{n-t}^2 to be the set of all permutations π^2 of $N_2=\{t+1,\dots,n\}$. Then the PROP can be mathematically stated as

$$\min_{\pi^1\in\Pi_t^1,\pi^2\in\Pi_{n-t}^2}\left\{\left(\sum_{i,j\in N_1; i<j} c_{ij} d_{ij}^{\pi^1}\right)+\left(\sum_{i,j\in N_2; i<j} c_{ij} d_{ij}^{\pi^2}\right)\right.$$

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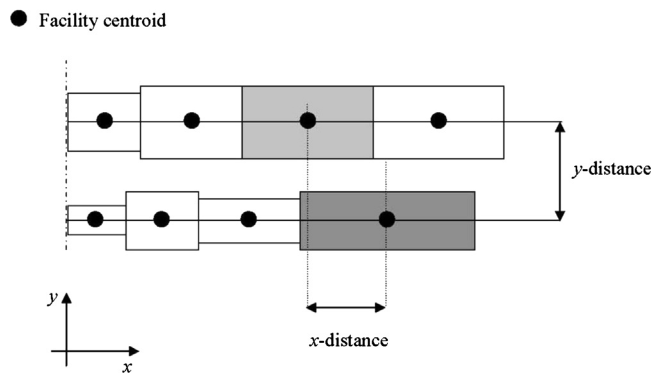

Fig. 1. Arrangement of facilities along two parallel straight lines on a floor plan. The distance between the grayed facilities is illustrated (as the ordering of facilities occurs in x, only the x-distances between facilities will have to be determined).

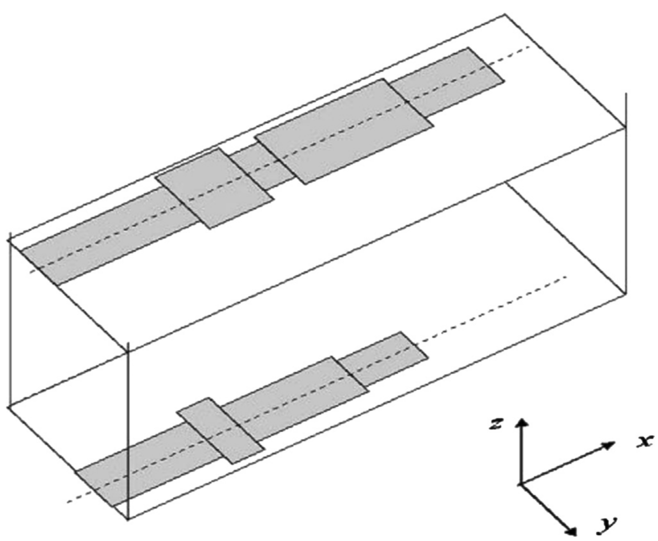

Fig. 2. The PROP on a multi-floor building. The orderings occur along the x-axis, thus only x-distances between facilities need to be determined. Other components of the distance remain constant.

$$+\left(\sum_{i\in N_1, j\in N_2; i<j} c_{ij} d_{ij}^{\pi^1,\pi^2}\right)\Bigg\} \quad (2)$$

where $d_{ij}^{\pi^1}$ is the x-distance between facilities i, $j\in N_1$ with respect to a permutation π^1; $d_{ij}^{\pi^2}$ is the x-distance between facilities i, $j\in N_2$ with respect to a permutation π^2; and $d_{ij}^{\pi^1,\pi^2}$ is the x-distance between facility $i\in N_1$ and facility $j\in N_2$ with respect to the permutations π^1 and π^2.

Note that if $t=n$, then formulation (2) reduces to the SRFLP formulation (1).

Facility layout problems are generally NP-hard. In this paper, a mixed integer programming (MIP) formulation of the PROP is presented, which extends a MIP formulation of the SRFLP given in [1]. We show that a PROP with n facilities may be solved faster than a SRFLP with n facilities. Such a result can help practitioners choose a layout type that meets their own particular needs and optimize it within available computational resources. Throughout the paper we assume $n\geq 3$.

2. Literature review

Several studies formulated the multi-row layout problem with facilities of equal sizes as a quadratic assignment problem (QAP). Heragu and Kusiak [23] presented three formulations. One of these was for the multi-row layout problem in which facilities are of equal area. The other two were for the layout problem with facilities of unequal area.

The multi-row layout problem has many applications such as computer backboard wiring [31], campus planning [17], scheduling [21], typewriter keyboard design [27], hospital layout [18], the layout of machines in an automated manufacturing system [22], balancing hydraulic turbine runners [25], numerical analysis [12], optimal digital signal processors memory layout generation [32].

The single-row facility layout problem (SRFLP) was first studied by Simmons [29]. Its particular case in which all facilities have equal length is known as the minimum linear arrangement problem (e.g. [3]). SRFLP applications include the assignment of files to disk cylinders, the arrangement of books in a shelf, warehouse layout [26]; and the layout of machines on one side of the straight path of an automated guided vehicle (AGV) [22]. SRFLP research has been very active and has produced different *exact methods* (e.g. [1,2,4,7,9,24]), *heuristics* (e.g. [15,16,22,28,30]) or *lower bounds* (e.g. [7,8,10,24]). In health-care settings a single row layout may be required. For infection control, a single-row ward layout works better than a double-row layout with a central corridor in terms of natural ventilation and daylight (e.g. [11]).

In the double-row layout problem (DRLP) facilities are arranged along two parallel rows but – differently from the PROP – no facilities are restricted to any given row. Another difference between the DRLP and the PROP is that the PROP assumes that the arrangements in both rows start from a common point and that no space is allowed between two adjacent facilities, while the DRLP does not make such assumptions. Moreover, the DRLP assumes that the distance between the two parallel rows is zero, while the PROP does not. The DRLP has been studied by Amaral [6], Chung and Tanchoco [14], Heragu and Kusiak [22]. For the machine layout application, a clearance between each pair of machines was considered by some authors (e.g. [14,22]). However, in many situations, the clearance can be included in the dimensions of the machines (e.g. [6]). For some applications the clearances are insignificant (e.g. room arrangement along a corridor [5]) and can be ignored.

Different extensions of the multi-row layout problem have appeared in the literature. For example, Gen et al. [20] formulate a multi-row machine layout problem where there is a clearance given as a fuzzy set between any two adjacent machines. They use Genetic algorithms as a heuristic technique for solving the problem. Ficko et al. [19] considered the design of a flexible manufacturing system (FMS) in one or multiple rows. The most favorable number of rows and the sequence of facilities in each individual row are found using genetic algorithms. In their work, the greatest length of a row (a) is an input parameter. The number of facilities in one row is limited using the parameter a. They use the following constructive procedure to determine the arrangement into rows: place facilities, one by one, at a row and when the length of the arrangement at that row becomes greater than a, the next facility is placed into a new row. This procedure repeats, until all facilities have been arranged into rows. They also describe their coding of "organisms", fitness function and genetic operations.

3. The SRFLP formulation of Amaral [1]

Consider the following:

- A vector $\alpha=(\alpha_{i,j})_{(1\leq i<j\leq n)}$ such that $\alpha_{i,j}=1$ when facility i is to the left of facility j and $\alpha_{i,j}=0$ otherwise.
- A vector $d=(d_{i,j})_{(1\leq i<j\leq n)}$ such that $d_{i,j}$ represents the distance between the centers of facilities i and j.