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Full Length Article

Large deflection analysis of curved beam problem with varying curvature and moving boundaries

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ABSTRACT

The paper presents experimental and theoretical large deflection analysis of non-uniformly curved beam with moving boundaries under static loading within elastic domain. A master leaf spring is considered as physical model of the curved beam problem and its load–deflection behaviour is studied experimentally in a specially designed testing rig. Beside direct deflection measurement at some discrete points within the specimen domain, image processing technique is also used to obtain complete deflection profiles under loaded conditions. The indirect deflection measurement through post processing photographs of loaded master leaf is implemented manually in AutoCAD®. Deflection behaviour of the physical system involves strong geometric nonlinearity coming from non-uniform initial curvature, moving boundaries, nonlinear kinematics due to coupling between bending, stretching, shear deformation and large deflection, asymmetry in beam geometry and eccentricity in load application point with respect to geometric centre. All of these complicating effects are considered in the mathematical model of the physical system. As large deflection involves a large rigid body motion and the induced strain coming from deformation displacement is rather small, present analysis is carried out within elastic regime where material constitutive relation remains linear. Hence system governing equation is derived within the framework of geometric nonlinearity and small strain assumption, using energy principle based variational method. The nonlinear governing equation, in association with complicated moving boundary conditions, is solved iteratively through incremental loading using an updated Lagrangian approach. After each incremental load step, kinetic relation is also satisfied through shear force balance. Numerical results are generated for the same loading conditions of the experimental work and comparisons between theoretical and experimental results are quite good. However, the comparison study leads to identification of several geometric parameters of the physical system, incorporation of which may provide more realistic simulation.

1. Introduction

Various structures and machine elements in civil, aerospace and mechanical engineering disciplines are generally modeled as beam. Most of such practical members are initially curved and show nonlinearity in their deformation behaviour. Hence precise design of such structural elements calls for nonlinear analysis. Nonlinearities in beam bending problem are generally manifested through nonlinear kinematic and material constitutive models and they are known as geometric and material nonlinearity. Large deformation of flexible members induces a large rigid body motion and small strain. Hence linear material modeling is generally used for large deflection analysis of such slender structures within elastic limit. The present large deflection problem only focuses on nonlinearities associated with nonlinear kinematic and kinetic relations, and material nonlinearity is out of scope of the present paper. Thus relevant research papers regarding geometric nonlinear analysis of beam and equivalent structures like leaf spring, arch, compliant mechanisms, etc., are critically reviewed and several observations are presented in the following paragraphs.

Trivial solution of beam bending problem generally linearizes Euler-Bernoulli moment-curvature relation assuming small deflection and hence cannot be used for beam undergoing large deflection. This produces the basic difference between small and large deflection analyses. One of the classical approaches, widely used to large deflection beam bending problem, is elliptic integral approach which provides solutions for nonlinear displacement field in terms of elliptic integrals [1–3]. This analytical approach

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becomes inconsistent when generalized loading condition is considered, even for an initially straight uniform beam [4,5]. In addition, spatial variation of beam stiffness along length enforces difficulty. In such cases, classical mechanics based system governing equation is solved through series approximation for unknown field [6–9] or using some iterative shooting process [3,10–12], depending on the strength of nonlinearity present in governing equation. For inextensible beam, iterative process is generally converged with the constant beam length. Constancy in beam length remains effective for deformation under follower loading [13,14]. However, conservative loading results some centre line stretching and in addition, presence of multiple solutions makes iterative approaches unsuitable to undertake more complicated problems [15]. Considerable stretching effect is generally encountered for beams with high slenderness ratio, whereas, shear deformation becomes significant for stub beams. Shear deformable beam is generally analyzed in the frame work of Timoshenko beam model and higher order shear deformation theories incorporating warping of beam cross-section [16,17]. Generally, combined effect of stretching and shear deformation is not considered in large deflection analysis. However, actual deformation process of a flexible structural member is vastly complicated as it includes combined effects of bending, stretching, shear deformation, torsion, warping of beam cross-section, and many such other complicated phenomena. In addition, most of the natural and manmade structures are arbitrarily curved in space which increases complexity of a problem [18]. Rigorous analysis of such three dimensional deformation characteristics of generalized naturally curved beam like structures tends towards geometrically exact beam theory, popularly known as Simo-Reissner beam theory [19]. An extensive review on development of geometric nonlinear beam theory is well documented in a recently published review paper [20].

Complicating effects, generally encountered in large deflection beam problem, arise from several problem parameters like loading condition, boundary condition, initial geometry, etc. Complications associated with loading pattern generally include change in intensity of distributed load in the course of large deformation and presence of singularity point within problem domain [4,5,11,21]. Nonlinear effects, predominantly coming from boundary condition, include development of friction force at simple supports [2] and elastic restraints of boundaries [3,22]. The friction force generated at supports and in addition, initial curvature of beam geometry and coupled transverse-in-plane displacement field add more complexity in nonlinear system response [6,11,23,24]. When such complicating effects are considered, even to some extent, classical mechanics based approach becomes inappropriate and analysis is carried out in the framework of variational mechanics [5,21,25–30]. In this approach, governing equation is derived through minimization of error arising from force [5,25,26,30] or energy [21,27–30] balance. Solution of variational equation, considering whole problem domain leads to semi analytical method [5,21], whereas solution through domain decomposition leads to a pure numerical method, e.g., finite element method [23,27–29]. Due to strong dependency of system response on deformation state, change in geometry greatly influences deformation characteristics of beam with complicated geometry, undergoing very large rotation and translation. Geometry updation is sometimes implemented considering initial configuration as reference, which leads to total Lagrangian approach [31–33]. Whereas, solution with last calculated geometry during incremental loading as reference provides more realistic prediction of system behaviour and known as updated Lagrangian approach [33,34].

In spite of numerous theoretical research works on large deflection of beam, experimental works are rarely reported. Most of the reported experimental works are performed with several beam [31,35,36] and equivalent structures like leaf spring [11,37,38], arch [39], etc., to validate theoretical models. In most of the experimental work, deflection is only measured at some specified points within physical domain, generally at load application point, using several precise instruments [31,35] and displacement sensors [9,21,37,40]. Whereas measurement of complete deflection profile is also reported in a research paper [11], where deflection measurement involves image processing technique.

Large deflection analysis of slender beam having initial straight profile, with classical boundary condition has been reported in a large number. Analysis of slender beams with initial uniform curvature is also reported in a moderate number, by modeling most of them as circular segment. Whereas geometric nonlinear analysis of extensible and shear deformable beam undergoing large deflection with generalized non-uniform initial curvature, asymmetric geometry, eccentric loading and moving boundaries is rare. Hence large deflection behaviour of a master leaf spring is simulated experimentally and theoretically as it involves all the mentioned complicating effects. Experiment is performed in a specially designed three point bending set-up. Several physical parameters of the system are identified and incorporated in the mathematical model and analysis is carried out through energy principle based geometry updation technique.

2. Experiment

As mentioned earlier, the present work simulates load-deflection behaviour of asymmetric non-uniformly curved beam with moving boundaries under eccentric static loading. A master leaf spring is considered as specimen of the present experimental work whose geometry under no-load condition is shown in Fig. 1 along with some major dimensions. Complete profile of the specimen is obtained in a Cartesian frame (X_e^0, Y_e^0) by measuring coordinates of twenty-one points, equally spaced along X_e^0 axis (marked as 2 to 22 in Fig. 1). The two end points of measurement domain are marked as points 1 and 23. These end points of curved beam domain (A_1, B_1) are defined as points of tangential intersections of master leaf centre line with pitch circles of the eye ends. Midpoint of the straight line joining the eye centres (span) is considered as origin of Cartesian coordinate system (X_e^0, Y_e^0) in the present geometry measurement and complete profile of the master leaf is presented numerically in Table 1. It is obvious from Fig. 1 and the tabulated values in Table 1 that no-load profile of the master leaf is asymmetric and have non-uniform curvature throughout the domain. In addition, centre line of hole present in the specimen (refer Supplementary Fig. S1 for enlarged view of the hole) has an eccentricity of 1.6 mm with respect to the origin of (X_e^0, Y_e^0), which will produce eccentric loading as described later on in the following paragraphs.

To obtain deflection characteristics of the above specified master leaf spring under static load, experiment is carried out on a specially designed experimental set-up. Photograph and schematic diagram of the set-up are shown in Fig. 2(a) and (b) respectively. Main components of the set-up fall into two categories and they are support structures (items 3, 5, 6, 10, 11) and load imparting components. Load imparting components mainly consist of load connector (item 7), vertical guide rod (item 1) and bush (item 2). Detailed drawings of the components are not presented separately to maintain compactness of the paper. Eye ends of the master leaf spring are assembled with roller support sub-assemblies (item 9), and this roller ended master leaf spring is placed on the C.I. bed (item 10) of the test rig underneath the load connector. However before placing the master leaf in the test rig, twenty-one equally spaced points are marked on the master leaf spring along its centre line, using prick punch (refer Supplementary Fig. S1 for clear understanding of the markings). These marked points on the