ARTICLE IN PRESS

Engineering Science and Technology, an International Journal xxx (2017) xxx–xxx

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Contents lists available at ScienceDirect

Engineering Science and Technology, an International Journal

journal homepage: www.elsevier.com/locate/jestch

JESTECH

Full Length Article

The application of ant colony optimization in the solution of 3D traveling salesman problem on a sphere

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A R T I C L E I N F O

Article history:
Received 7 July 2017
Revised 18 August 2017
Accepted 23 August 2017
Available online xxxx

Keywords:
Ant colony optimization
Metaheuristic
Spherical geometry
Max-Min Ant System
nonEuclidean TSP

A B S T R A C T

Traveling Salesman Problem (TSP) is a problem in combinatorial optimization that should be solved by a salesperson who has to travel all cities at the minimum cost (minimum route) and return to the starting city (node). Todays, to resolve the minimum cost of this problem, many optimization algorithms have been used. The major ones are these metaheuristic algorithms. In this study, one of the metaheuristic methods, Ant Colony Optimization (*ACO*) method (Max-Min Ant System – *MMAS*), was used to solve the *Non-Euclidean* TSP, which consisted of sets of different count points coincidentally located on the surface of a sphere. In this study seven point sets were used which have different point count. The performance of the *MMAS* method solving Non-Euclidean TSP problem was demonstrated by different experiments. Also, the results produced by *ACO* are compared with Discrete Cuckoo Search Algorithm (*DCS*) and Genetic Algorithm (*GA*) that are in the literature. The experiments for *TSP* on a sphere, show that *ACO's* average results were better than the *GA's* average results and also best results of *ACO* successful than the *DCS*.

1. Introduction

Traveling salesman problem (*TSP*) is the problem of a salesman who needs to visit all the cities in the schedule and return to the starting point by spending less. One of the parameters such as path, time, cost and path in *TSP*, can be optimized. *TSP* is called also as a Hamiltonian path problem that is used in computer science for data modeling. The *TSP's*, evaluated in discrete and combinatorial problems has been comprehensively studied in the field of similar graph theory problems. *TSP* is considered in two categories as symmetric and asymmetric. In the symmetric *TSP*, always, the distance between x. city and y. city is equal, i.e., $d_{xy} = d_{yx}$. In the asymmetric *TSP*, the distance matrix between cities may not be the equal for all cities.

In order to solve *TSP*, many methods have been developed. They are divided into two groups as heuristics and exact methods in terms of obtaining the optimal results. Branch-and-bound, branch-and-cut and iterative improvement are the exact solution methods for *TSP* [4,23]. Various heuristic algorithms, based on Simulated Annealing [21,12], Genetic Algorithms (*GA*) [35,16,18,38,26]), Tabu Search [14,15,25], Artificial Neural Networks [19,22,30,28] and Ant Colony Systems [2,3,5,13,7,8,33,32,1] have been developed which make the closest possible solutions to the best solutions at a reasonable time. In the meantime, to solve *TSP*, 2-opt, 3-opt and 4-opt local search algorithms were also used [20]. Some researchers to make optimum results of *TSP*, have studied hybrid evolution algorithms [24,39,27,34,25]. Some *TSP* applications were executed on the basic 3D geometric figures like spheres and cuboids [36,37,31,29,10,9,11]. An algorithm was proposed by making the solution of *TSP* with *GA* on a cuboid [36] and a sphere [37]. In [31], the particle swarm optimization algorithm (*PSO*) was proposed by making the solution of *TSP* on cuboid. An algorithm was proposed by making the solution of *spherical TSP* with Cuckoo Search algorithms on a sphere [29]. And also, algorithms were proposed by making the solution of *sphericalTSP* and *cuboidTSP* with ACO and PSO on a sphere and cuboid [9].

One of the metaheuristic algorithms, Ant colony optimization (*ACO*), used to solve discrete optimization problems, was proposed by Marco Dorigo in 1992 as a PhD thesis [5]. *ACO* is a metaheuristic computational algorithm technique. *ACO* was used to solve graph problems by investigating possible paths on the graphs. *ACO* is inspired by the behavior of ants that provides to find shortest distance between their nest and food resource by means of pheromone.

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Peer review under responsibility of Karabuk University.

http://dx.doi.org/10.1016/j.jestch.2017.08.005
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Ants choose shortest way while searching food resources rapidly in progress of time. Various *TSP* applications have been successfully solved with *ACO* techniques.

MAX-MIN Ant System *(MMAS)* that is an improvement over the Ant System *(AS)* proposed by Stützle and Hoos [32]. *MMAS* differs from AS at pheromone update. In AS, when complete the tours, each of ants updates their pheromone trials. But in *MMAS* just the best ant updates the pheromone trials and pheromone level is bounded between minimum-maximum limit.

In this study, *TSP* was solved for the points on a sphere by *ACO* algorithm (*MMAS*). To our knowledge, so far there is no study solving *TSP* by this technique in 3D. For the available *TSP*s, the coordinates of the points and the distances between them are known. Since all the points are present on a sphere and passage from one point to the other is carried out from the sphere surface, this problem is different from the existing *TSP*s. The study covers the definition of points on a sphere, finding the distances between the points and adaptation of the problem to the *ACO*.

2. The basic of a sphere

A sphere is a 3D object made up of points that are at the same distance from a given point in space. Every point (with coordinates of *x, y, z*) distributed at an equal distance *r* from the center is located on the sphere surface. In other words, a sphere is obtained by turning of an arc, drawn at a same distance from the origin with coordinates of *x-y*, around the z-axis. The relation between the *x, y, z* coordinates and the radius of a sphere is formulated by the Eq. (1):

$$\mathrm{r} = \sqrt{\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2} \quad (1)$$

The radius of a sphere, *r* is the distance from the center (point A) to the points on the sphere (B, C, D and E) and shown in Fig. 1. Every point on the sphere has coordinates of *x, y, z* and these values always satisfy the Eq. (1).

When a problem is considered on a sphere, the first example that comes to mind is the geometric similarity of the Earth to a sphere. The circle passing throughout the sphere center and bounded by a sphere is big circle called equator of the Earth. This circle becomes important when the minimum distance between two points, i.e., geodesic, on a sphere along the lower cross-section is considered. The curves are called as geodesics on any surface of sphere that minimize the distances between their points [37].

2.1. Mathematical notation of points on a sphere

Euclidean curves have a single dimension. These curves can be defined by a single parameter called *u* along a 3D curve. In other words, in terms of parameter *u* points out the Cartesian coordinates. Any point on a curve can be represented by a point vector function according to the given reference Cartesian coordinates [17]:

$$\mathrm{P}(u) = (x(u), y(u), z(u)) \quad (2)$$

Generally, coordinate equations can be set up in a way that where parameter *u* is described between *0* and *1*. As an example, a circle on the xy-plane centered at the origin is defined in a parametric form given below [17]:

$$x(u) = \mathrm{r}.\cos(2\pi u) \quad \mathrm{y}(u) = \mathrm{r}.\sin(2\pi u) \quad z(u) = 0, \quad 0 \leqslant u \leqslant 1 \quad (3)$$

Circles and circular curves can also be defined in other parametric forms. Sloping Euclidean surfaces are two-dimensional varieties described by parameters *u* and *v*. A coordinate position on a surface can be represented by a parameterized vector function with *u* and *v* parameters for the coordinate values of *x*, *y* and *z* [17].

$$\mathrm{P}(u) = (x(u, v), y(u, v), z(u, v)) \quad (4)$$

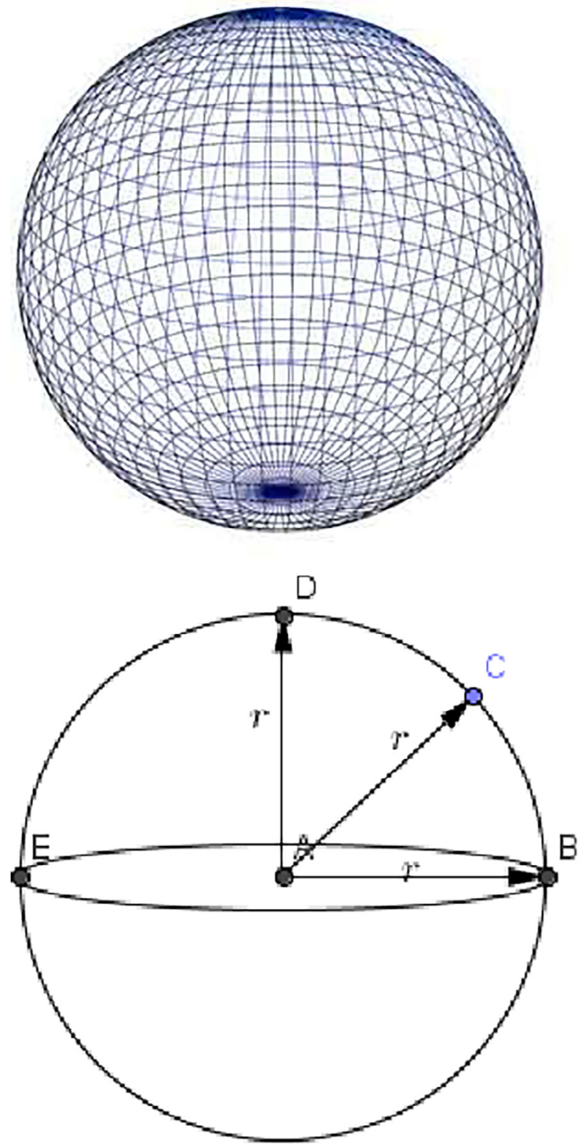

Fig. 1. Spherical surface and the radius.

Each of Cartesian coordinate values is a function of surface parameters *u* and *v*, which change between *0* and *1*. The coordinates of a spherical surface centered at the origin with a radius *r* can be defined by the Eq. (5) [17]:

$$\begin{aligned} x(u, v) &= \mathrm{r}.\cos(2\pi u).\sin(\pi v) \\ y(u, v) &= \mathrm{r}.\sin(2\pi u).\sin(\pi v) \\ z(u, v) &= \mathrm{r}.\cos(\pi v) \end{aligned} \quad (5)$$

where, the parameters *u* and *v* define the constant lines of latitude and constant lines of longitudes on the surface, respectively [17]. To illustrate, *x, y, z* coordinates for the different values of parameters *u* and *v* were calculated according to the Eq. (5) and are given in Table 1. Note that *r* was taken as *1* and upon the increase in value of *r*, the values of *x, y, z* should also be increased in the same manner.

2.2. Finding the shortest distance between all pairs of points on the surface of unit sphere

On a spherical surface, minimum distance between two points (P_1, P_2) is along the arc of a great circle (Fig. 2). So, in radians, the

Please cite this article in press as: H. Eldem, E. Ülker, The application of ant colony optimization in the solution of 3D traveling salesman problem on a sphere, Eng. Sci. Tech., Int. J. (2017), http://dx.doi.org/10.1016/j.jestch.2017.08.005