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Review

Compressed sensing trends in magnetic resonance imaging

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A R T I C L E I N F O

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A B S T R A C T

The theory of Compressive Sensing (CS) has experienced a tremendous growth through continuous works of researchers from different cross platform domains of study. The strict realm of Shannon-Nyquist sampling theorem is compromised and an image can be reconstructed from fewer measurements than it was shown necessary to be, but with a trade-off in the efficiency. In biomedical signal processing, especially Magnetic Resonance Imaging (MRI), the potential applicability of CS is long observed. Since then quite a large number of research work in this field has been proposed, a few with experimental analysis, which establish its applicability in the domain of MRI. Since the topic is too broad, this review paper presents a discussion and summary of important works on different fields of CS-MRI. The challenges, limitations and advantages of different techniques of CS-MRI are studied and future trend/ direction is predicted.

Contents

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1. Introduction

Magnetic Resonance imaging (MRI) is a non evasive radiology technique which uses magnetic fields and radio waves to produce images of the human body. Generally data points in MRI are complex in frequency domain with magnitude and phase components. These constitute a matrix called k-space. Some of the other parameters that influence data acquisition of MRI are – longitudinal relaxation time $T1$ and transverse relaxation time $T2$ which vary from tissue to tissue. However traditional sampling of continuum data points has high sampling rates, generating a huge number of samples. This requires considerable scan time which makes MRI to be a slow data acquisition system. Hence over more than two decades, attempts at lowering data acquisition time have been made by researchers. In this direction, Compressive Sensing (CS) has emerged as a promising solution. In CS it is possible to reconstruct an image from fewer measurements than required in traditional sampling provided some constraints are satisfied. The methods and trends in CS have undergone a massive improvement over last few years such as parallel CS data acquisition MRI, dictionary learning and motion estimation techniques in dynamic MRI. They are practically implemented with better results.

2. Basic underlying CS principles

The CS employs the concept of random under-sampling which may reduce the number of k-space samples to be measured during data acquisition in an MRI machine and hence reduce scan time. However there are some basic requirements of CS for a reconstruction to be optimal.

The first condition for application of CS theory is that the signal should be sparse in some transform domain. The sparsity of a signal in mathematical terms can be defined as: Let us suppose, we have a discreet time signal x in $\mathbb{R}^N$ which can be expressed in terms of a set of orthogonal basis or support of vectors $[\Psi_i]_{i=1}^{N}$ as follows: $x = \sum_{i=1}^{N} s_i \Psi_i$, where s_i is the coefficient sequence of x. In matrix form, we can simply write the above equation as $x = \Psi s$. Then, signal x is said to be k-sparse if only k entries of s are non-zeroes and remaining $(n - k)$ entries are zero. The abstract CS theory [1,2,52,53], suggests that given some conditions and constraints, it is possible to almost exactly reconstruct a given sparse signal (its support has cardinality less than or equal to k from a small number of available, random linear combinations of the signal by a non-linear reconstruction strategy. In order to measure all N coefficients of x, we consider a vector y of dimension $M \times 1 (M < N)$ such that

$$y = \phi x \tag{1}$$

where $\{\phi_j\}_{j=1}^{M}$ is a $M \times N$ matrix or collection of vectors called measurement matrix with ϕ_j^T as rows. Substituting the value of x from (1) we can write,

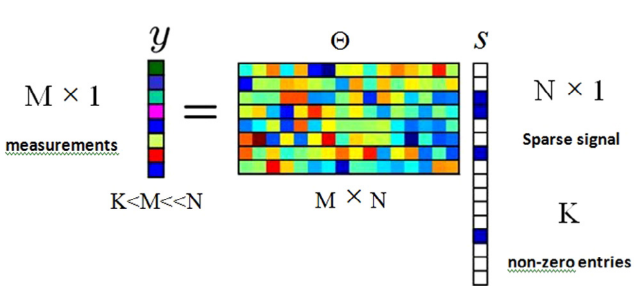

Fig. 1. The basic CS principle.

$$y = \phi x = \phi \Psi s = \Theta s \tag{2}$$

where $\Theta = \phi \Psi$ is a $M \times N$ matrix. In this way, we can transform a $N \times 1$ k-sparse signal into $M \times 1$ set of measurements y, by using a matrix Θ as shown in Fig. 1. However, for faithful reproduction of the signal x, the matrix ϕ and reconstruction strategy adopted, must satisfy certain properties.

2.1. Restricted isometry property

In Eq. (2) the matrix ϕ must map two different signals into two different sets of measurements. Hence all column sub-matrices of ϕ must be well contained. Candes, Romberg and Tao [1,3] proposed that the sampling matrix ϕ must satisfy the following condition: For a given constant δ_{2k} , the condition-

$$(1 - \delta_{2k})||x_1 - x_2||_2^2 \leqslant ||\phi x_1 - \phi x_2||_2^2 \leqslant (1 + \delta_{2k})||x_1 - x_2||_2^2 \tag{3}$$

must hold for all k-sparse vectors x_1 and x_2. The property is called restricted isometry property (RIP) and the constant δ_{2k} is called restricted isometry constant. The property states that all pairwise distances must be well preserved in measurement matrix ϕ. Though it is computationally difficult to check whether a particular matrix satisfies the above mentioned property, it has been found that many types of random matrices (example: independent and identically distributed Gaussian measurement matrix) satisfies the RIP.

2.2. Incoherence

Since under-sampling will result in aliasing of data points, the behavior of the aliasing artifacts must be incoherent (noise like) in the transform domain. In case of the under-sampling being not random, it is impossible to distinguish between signal and its aliases.

2.3. Reconstruction strategy

The reconstruction strategy must aim for a solution $x \in \mathbb{R}^N$ from system of equation stated in Eq. (2). Since the system of equation is under determined, infinitely many solutions exist for the same set of measurements. However given the condition that x is sparse and measurement matrix satisfies RIP, we can exactly recover x by solving a l_0-minimization problem,

$$\min_{\tilde{x} \in \mathbb{R}^N} \|\tilde{x}\|_{l_0} \text{ subjected to } \phi\tilde{x} = y \tag{4}$$

This minimization problem was shown to be NP-hard [4,5]. A solution to the minimization problem is given by using l_1 norm instead of l_0 norm which may yield similar result and the new strategy is that if

$$\|\tilde{x}\|_{l_1} = \sum_{i=1}^{N} |\tilde{x}|, \text{ then, } \min_{\tilde{x} \in \mathbb{R}^N} \|\tilde{x}\|_{l_1} \text{ subjected to } \phi\tilde{x} = y \tag{5}$$

In a foundation work of Compressive sensing, Candes and Tao [1] explained that the signal reconstruction strategy should be

$$\hat{x} = \arg\min \|y - \phi x\|_{l_2}^2 + \lambda \|\Psi x\|_{l_1} \tag{6}$$

where, x and $\hat{x}$ are signal of interest and reconstructed signal respectively, ϕ is the acquisition matrix of size $N \times M, y$ is the captured data and Ψ is the sparsifying transform and λ is the regularization parameter. The objective function here is the l_1-norm minimization and the l_2-norm constraint enforces data consistency. In words, out of all potential sparse solutions, the equation selects one solution that is compressible. It has been well established fact that the proper norm for image restoration is the Total variation (TV) norm and not the l_2 norm. The TV norms are essentially l_1