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Connected Cubic Network Graph

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ABSTRACT

Hypercube is a popular interconnection network. Due to the popularity of hypercube, more researchers pay a great effort to develop the different variants of hypercube. In this paper, we have proposed a variant of hypercube which is called as "Connected Cubic Network Graphs", and have investigated the Hamilton-like properties of Connected Cubic Network Graphs (CCNG). Firstly, we defined CCNG and showed the characteristic analyses of CCNG. Then, we showed that the CCNG has the properties of Hamilton graph, and can be labeled using a Gray coding based recursive algorithm. Finally, we gave the comparison results, a routing algorithm and a bitonic sort algorithm for CCNG. In case of sparsity and cost, CCNG is better than Hypercube.

1. Introduction

The most common network topologies used in practice are trees, cycles, grids, tori, meshes and hypercubes. In this paper, in order to present a new perspective for network topologies, we used hypercube topology for construction of a hypercube variant called Connected Cubic Network Graph (CCNG). The $CCNG(k,m,n)$ of n nodes in x dimension, m nodes in y dimension, k nodes in z dimension was defined and constructed in current study. $CCNG(k,m,n)$ is a recursive graph which can be constructed by two ways. The first way, a finite number of $H(n)$ are connected in $3D$ space, in which $H(n)$ characterizes a cube in this space, and the second way is to connect a finite number of the mesh structures obtained in the first way. Here, firstly, we will connect a finite number of cubes $(H(3))$ in a fashion that each cube has a common surface with the subsequent cube. Further, we will connect a finite number of hypercube $H(4)$. ($H(4)$ is a hypercube in $4D$ space) in a fashion that each hypercube has a common cube. Furthermore, we will connect a finite number of this mesh structures created by connecting cubes to be a surface in common. Thus, a Hamilton graph is obtained by connecting hypercubes of dimension 3 or 4.

In this paper, after obtaining $CCNG(k,m,n)$ graph, we have investigated the Hamilton-like properties of $CCNG(k,m,n)$.

The rest of this paper is organized as follows. Section 3 describes the definitions of $CCNG(k,m,n)$ in $3D$ space. Section 4 shows the construction and characteristic analyses of $CCNG(k,m,n)$. The recursive algorithm used for labeling the nodes of mesh structure is given in Section 5. In Section 6, we give comparison results. In Section 7, we give a routing algorithm for $CCNG$. In Section 8, the bitonic sort algorithm is mapped to $CCNG(k,m,n)$. Finally, we give a conclusion.

2. Background and related work

An interconnection network is usually represented by a graph $G=(V,E)$, which is a pair of sets where V represents the set of nodes and E represents the set of edges. Hypercube graph $H(n)$ has been widely used for interconnection networks. Hypercube graph $H(n)$ is a recursively definable graph. $H(n)$ has nice properties such as regular graph, simple node labeling, good connectivity, symmetry and cost etc. $H(n)$ of dimension n connects up to 2^n nodes, each of which can be labeled by n-bit address uniquely, using a direct connection between two nodes, if and only if their n-bit addresses differ in exactly one bit position. The reason for the popularity of the $H(n)$ can be attributed to its topological properties, the ability to use simple routing algorithms and the ability to permit the embedding of commonly-rewired interconnection patterns. In recent years, many properties of hypercubes of dimension n have been studied, extensively (cf. the survey [12] and [6,9,10,22,23]). Also, hypercube variants have been derived to ensure the performance increasing of hypercubes. Some of the

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most popular hypercube variants are the Folded Hypercube [3,5], the Crossed Cube (Twisted Cube) [8,11,19] and the Hierarchical Cubic Network (cf. the survey [1] and [7,14,15]). The aim of these approaches is just to reduce the diameter of the hypercube but reserve its advantages. Recently, Karci and Selcuk [16] defined a new hypercube variant called "Fractal Cubic Network Graph" using a fractal structure. They investigated the Hamilton-like properties of a new hypercube variant. Besides, some authors have proposed new interconnection networks. Interested readers can refer to ref [2,20,21] for details.

Throughout this paper, "||" denotes the concatenation of two strings. The Hamming distance is calculated with $\sum_{i=0}^{n-1}(a_i \oplus b_i)$, where summation is equal to summation of $a_i \oplus b_i$ (bitwise-XOR operation). $H(3)$ is called a cube in $3D$ space and $H(4)$ is called a hypercube in $4D$ space. Let us remember three-dimensional coordinate plane as seen in Fig. 1.

3. The construction of connected cubic network graphs (CCNG (k,m,n))

In this section, we gave the definition of $CCNG(k,m,n)$. For the reader's convenience, we first gave the following general definition:

Definition 1. ($CCNG(k,m,n)$): $CCNG(k,m,n)$s can be defined in three steps.

(a) $CCNG(k,m+1,n)$ is a cubic network graph which is obtained by connecting $\sum_{i=0}^{m}2^i$–cubes in a fashion that consecutive cubes have a common surface in xz plane. For example, the mesh structure given in Fig. 3 is a $CCNG(1,2,1)$.

(b) $CCNG(k,m+1,n+1)$ is a cubic network graph which is obtained by connecting the number of $\sum_{i=0}^{n}2^i$ the mesh structures, $CCNG(k,m+1,n+1)$, which have the lower and upper surfaces to be in common along with z axis. For example, mesh structure in Fig. 4 is $CCNG(1,2,2)$.

(c) $CCNG(k+1,m+1,n+1)$ is a cubic network graph which is obtained by connecting the number of $\sum_{i=0}^{k}2^i$ the mesh structure, $CCNG(k,m+1,n+1)$,which have the lower and upper surfaces to be in common along with x axis. For example, mesh structure in Fig. 5 is $CCNG(2,2,2)$.

Analogues of these definitions can be handled in different ways. For example,

$$CCNG(k,m+1,n) = CCNG(k+1,m,n) = CCNG(k,m,n+1)$$

or

$$CCNG(k+1,m+1,n) = CCNG(k+1,m,n+1) = CCNG(k,m+1,n+1).$$

Further, analogues of these definitions are replaced by $H(3)$ and $H(4)$.

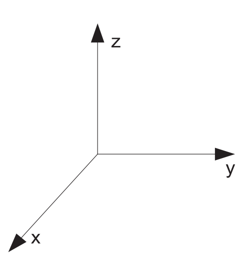

Fig. 1. Three-dimensional coordinate space.

Now, we can give a detailed definition by utilizing Definition 1 in [13]. In order to construct $CCNG(k,m,n)$ network, there are three cases to supply Definition 1 (a):

Case 1: $x = 2y$ and $y = z$
Case 2: $y = 2x$ and $x = z$
Case 3: $z = 2x$ and $x = y$

In Case 1, the number of nodes will be doubled in dimension x whereas the number of nodes in dimensions y and z will be remained constant. The same process can be applied to other dimensions (y and z). The construction of $CCNG(k,m,n)$ can be summarized as in Definition 2:

Definition 2. Assume that $CCNG(k,m,n)$ is Connected Cubic Network Graph. Two $CCNG(k,m,n)$s can be combined to construct a new network of size doubling the size of $CCNG(k,m,n) = G(V,E), k \geqslant 1, m \geqslant 1, n \geqslant 1$. There will be three cases:

(a) If doubling dimension is x, then the nodes and edges in $0||CCNG(k,m,n)$ and $1||CCNG(k,m,n)$ are also included in $CCNG(k+1,m,n) = G(V_x,E_x)$. If $\forall v_i \in V, p = 3,\ldots,k+m+n-1$, $2^p - 8 \leqslant Label(v_i) \leqslant 2^p - 5$, $|k-m| \leqslant 1, |k-n| \leqslant 1$ and $|m-n| \leqslant 1$, then $\forall(0||v_i, 1||v_i) \in E_x$.

(b) If doubling dimension is y, then the nodes and edges in $0||CCNG(k,m,n)$ and $1||CCNG(k,m,n)$ are also included in $CCNG(k,m+1,n) = G(V_y,E_y)$. If $\forall v_i \in V, Label(v_i)$ is even, $Label(v_i) < 2^{k+m+n-1}$, $|k-m| \leqslant 1, |k-n| \leqslant 1$ and $|m-n| \leqslant 1$, then $\forall(0||v_i, 1||v_i) \in E_y$.

(c) If doubling dimension is z, then the nodes and edges in $0||CCNG(k,m,n)$ and $1||CCNG(k,m,n)$ are also included in $CCNG(k,m,n+1) = G(V_z,E_z)$. If $\forall v_i \in V, Label(v_i) \in (\{4i|i = 0,1,2,\ldots,2^{k+m+n-2}\} \cup \{4i+1|i = 0,1,2,\ldots,2^{k+m+n-2}\})$, $|k-m| \leqslant 1, |k-n| \leqslant 1$ and $|m-n| \leqslant 1$, then $\forall(0||v_i, 1||v_i) \in E_z$.

Fig. 2 illustrates doubling process strategies and labeling techniques.

The Definition 2 uses $H(3)$ as a basic-building construction-block for constructing remaining $CCNG(k,m,n)$. This definition can be revised for using $H(4)$ as basic-building block.

4. Analytical properties of CCNG(k,m,n)

In this section, we showed the characteristic analyses of $CCNG(k,m,n)$. In Section 4.1, we focused on getting Hamilton properties of $CCNG(k,m+1,n)$. Similar of these process is replaced by $CCNG(k,m+1,n)$ and $\{CCNG(k+1,m,n)$ or $CCNG(k,m,n+1)\}$. In Section 4.2, we focused on getting Hamilton properties of $CCNG(k,m+1,n+1)$. Similarity of these process are replaced by $CCNG(k,m+1,n+1)$ and $\{CCNG(k+1,m+1,n)$ or $CCNG(k+1,m,n+1)\}$.

4.1. Hamilton properties of CCNG(k,m+1,n)

In this subsection, we found out analytical properties of $CCNG(k,m+1,n)$. Firstly, we gave a figure of mesh structure for $CCNG(k,m+1,n)$ where $k=1, m=1$ and $n=1$ from Definition 2 in Fig. 3. Also, it is easy to show that $CCNG(1,1,1)$ is same as $H(3)$. Then, $CCNG(1,1,1)$ is a Hamilton graph which is obtained by using 3 bit Gray Code.

Theorem 1. *Let's connect $\sum_{i=0}^{m}2^m$–cubes which have four nodes (a surface) to be in common, along with the y axis as Fig. 3. Thus, we have obtained Hamilton graph, which produced by traveling all nodes*