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Series analysis for the flow between two stretchable disks

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A B S T R A C T

In this paper, we present the semi-analytical/semi-numerical solution of an axis-symmetric flow between two coaxial infinite stretching disks. The governing momentum equations in cylindrical co-ordinates are reduced to fourth order nonlinear ordinary differential equation (NODE) with the relevant boundary conditions. The resulting nonlinear boundary value problem is solved by using Computer Extended Series Solution (CESS) and Homotopy Analysis Method (HAM). The effects of Reynolds number R and disk stretching parameter γ are discussed in detail. The resulting solutions are compared with the earlier numerical findings. The above methods admit a desired accuracy and the results are presented in the form of graphs. The validity of the series solution is extended to a much larger values of R up to infinity. Further, the variations of shear stress and pressure parameter as a functions of R and γ are analyzed. For very large R, the governing equation reduces to third order NODE with infinite boundary is solved by using Dirichlet series and the solution is compared with the numerical findings.

1. Introduction

The study of boundary layer flow of a viscous incompressible fluid over a moving boundary/stretching boundary has significant applications in engineering and industrial processes, such as plastic and metal industries involving surface stretching or extrusion processes [1–3]. Sakiadis [4,5] was the first researcher to propose the surface stretching problem based on boundary layer assumptions. According to Wang [6] it is not an exact solution of the Navier-Stokes (NS) equations. Crane [7] find an exact solution for the two dimensional stretching sheet problems, where the surface stretching velocity is proportional to the distance from the fixed slot. The generalization of Crane problem to a power law stretching velocity based on boundary layer flows discussed by Kukien [8] and Bank [9]. Gupta and Gupta [10] considered the Crane problem with mass injection/suction at the wall and Wang [11] studied the same problem in a rotating system. Wang [12] examined the three dimensional flow due to stretching flat surface. Brady and Acrivos [13] find another exact solution of NS equations involving the flow inside a channel or tube with a stretching wall. Wang [14] studied the flow outside an accelerating stretching tube, which was also demonstrated as an exact solution for the NS equations. The unsteady developments [15–17] and the spatial stability [18] of a class of similarity solutions for the Brady and Acrivos problem [13] were further studied. Zaturska and Bank [19] discussed the channel problem with combined effects of porous and stretching walls. Rasmussen [20] examined the steady viscous flow between porous disks with mass suction. Turkyilmazoglu [21–23], examined the MHD fluid flow and heat transfer, three dimensional MHD stagnation flow also discussed the flow and heat simultaneously induced due to a stretchable rotating disk. Mustafa et al. [24] analyzed on Bodewadt flow and heat transfer of nanofluids over a stretching stationary disk. Hayat and his collaborators [25–28] have discussed the various physical aspects of convection flow of carbon nanotubes with thermal radiation effects, partial slip effect and effects of homogeneous-heterogeneous reactions in flow of magnetic-Fe_3O_4 nanoparticles, and unsteady stagnation point flow of viscous fluid between rotating disks.

Recently Sheikholeslami and his associates [29–38] have examined the various semi-analytical methods such as DTM, ADM, HPM and OHAM for the solution of different types flow and heat transfer problems arise in fluid mechanics. In this paper, we present the series solution of the flow between two coaxial stretching disks for small and moderately large Reynolds numbers. In the first method, we investigate the flow problem based on a new type of series analysis (CESS) and present some interesting results. The salient features of this method are evidently explained by Van Dyke [39]. Bujurke and his associates [40–43] have clearly shown the potential applications of these methods in computational fluid dynamics. These methods reveal the analytical structure of the

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Nomenclature

d	distance between the disks	F, H	dimensionless stream functions
p	fluid pressure	η	similarity independent variable
r	radius of the disk	ν	kinematic viscosity of the fluid $(\mathrm{m^2\,s^{-1}})$
E	disk stretching strength parameter	ρ	density of the fluid $(\mathrm{kg\,m^{-3}})$
R	Reynolds number	γ	disk stretching parameter
u_r, u_z	velocity components in the directions r and z respectively	β	pressure parameter

solution which is not clear in case of other methods. The few manually calculated perturbation solutions in the low Reynolds number of the boundary value problem which enables us to propose a systematic series expansion to generate large number of universal polynomial coefficients by using recurrence relation and Mathematica. The resulting series will be limited in convergence by nonphysical singularities are extended to moderately high Reynolds numbers using an analytic continuation of the series solution. The location and nature of the singularity which restricts the convergence of the series is predicted using Domb-Sykes plot [44]. The analytic continuation can be achieved by extend the validity of the perturbation series to moderately larger values of Reynolds number using Pade' approximants. For large R, the governing NODE is reduced to third NODE with infinite interval and for the solution of this equation we have used the Dirichlet series method.

We also investigate the same flow problem using fast converging semi-analytical method called Homotopy analysis method (HAM) proposed by Liao [45]. The HAM provides the solution in much convenient way, to adjust and control convergence region of the series. In this method we have the liberty to choose base functions of the required solution and the corresponding auxiliary linear operator. Therefore, the HAM has an excellent flexibility and generality over all other analytical or approximate methods and also it is easy to use. Awati et al. [46] discussed the solution of MHD flow of viscous fluid between two parallel porous plates using CESS and HAM. Most recently, Hayat and his research colleagues [47–59], have successfully analyzed different types of fluids in MHD flow problems and stretching surfaces which arise in the engineering and science fields using HAM.

The paper is structured as follows. Section 1 describes the introduction; Section 2 develops the mathematical formulation of the proposed problem with relevant boundary conditions. The solution of the problem is obtained by Computer extended series as well as Homotopy analysis method in Sections 3 and 4 respectively. Section 5 presents results and discussion; Section 6 is about the conclusion (see Table 1).

2. Mathematical formulation

Consider an axis-symmetric viscous flow between two coaxial infinite stretching disks with a distance d between them. The disks are stretched in the radial direction with the velocity proportional to the radii and the lower disk is placed at the plane $z = 0$. The schematic diagram for the considered flow problem phenomenon is given in Fig. 1.

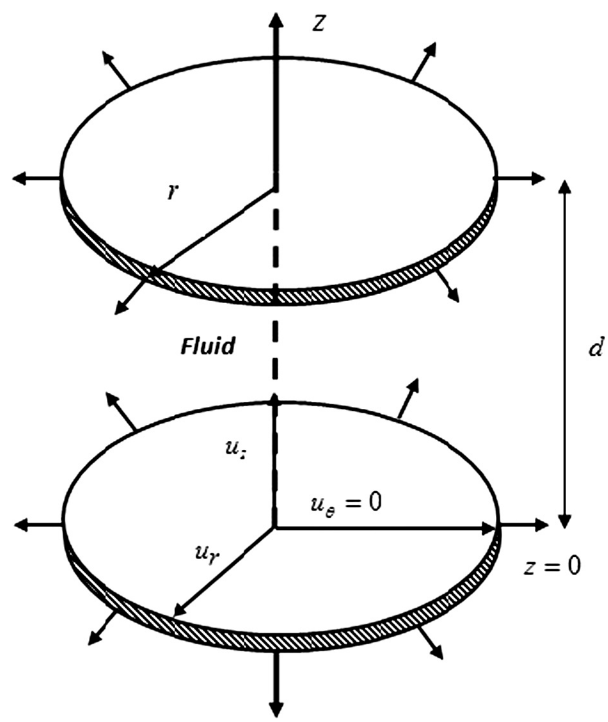

Fig. 1. Schematic diagram of the flow phenomenon.

For the viscous incompressible fluid in the absence of body forces and based on an axis-symmetry flow, the steady state NS equations in cylindrical polar coordinates [60] becomes

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{\partial u_z}{\partial z} = 0 \tag{2.1}$$

$$u_r\frac{\partial u_r}{\partial r} + u_z\frac{\partial u_r}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2}\right) \tag{2.2}$$

$$u_r\frac{\partial u_z}{\partial r} + u_z\frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{\partial^2 u_z}{\partial z^2}\right) \tag{2.3}$$

where $V = (u_r, u_z)$ is the velocity vector, ν is the kinematic viscosity of the fluid, p be the pressure of the fluid and ρ is the density of the fluid. We use the following transformations [61] such as

Table 1
Comparison of shear stress at the surface $f''(0), f(\infty)$ by using Dirichlet series with numerical method [61].

Dirichlet series				Numerical [61]	
a	γ	$f''(0)$	$f(\infty)$	$f''(0)$	$f(\infty)$
1.13583860	−1.50299405	−2.347441	−1.50299405	−2.347442	−1.502996

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