Engineering Science and Technology, an International Journal 20 (2017) 518–527

HOSTED BY

ELSEVIER

Contents lists available at ScienceDirect

Engineering Science and Technology, an International Journal

journal homepage: www.elsevier.com/locate/jestch

Full Length Article

Combined effect of heat source, porosity and thermal radiation on mixed convection flow in a vertical annulus: An exact solution

Michael O. Oni

Department of Mathematics, Ahmadu Bello University, Zaria, Nigeria

A R T I C L E I N F O

Article history:
Received 18 September 2016
Revised 5 December 2016
Accepted 25 December 2016
Available online 13 January 2017

Keywords:
Heat source
Thermal radiation
Mixed convection
Vertical annulus
Porous material
Exact solution

A B S T R A C T

This paper examines the effect of heat source, thermal radiation and porosity on mixed convection flow in a vertical annulus filled with porous material. The inner surface of outer cylinder is assumed to be the heated surface. Closed-form expression for temperature, velocity, Nusselt number, skin-friction and mass flow rate are obtained in terms of Bessel's function and modified Bessel's function of first and second kind. Based on depicted graphs, fluid temperature and Nusselt number increase with increase in radiation parameter and heat source parameter while velocity as well as skin-friction decreases with increase in radiation parameter and heat source parameter at the surfaces of the cylinder.

1. Introduction

Over the years, owing to the fact that cylinders have been used in nuclear waste disposal, energy extortion in catalytic beds and undergrounds, convective heat transfer about cylindrical geometries has begun to attract the attention of many researchers since some fluids are good emitter and absorber of thermal radiation, it is of interest to study the effect of heat source on temperature distributions and heat transfer when the fluid is capable of emitting and absorbing thermal radiation. This can be attributed to the fact that heat transfer by thermal radiation is becoming of greater importance when we are concerned with space applications (space journey) and higher operating temperatures.

Several investigations have been carried out on problem of heat transfer by radiation as an important application of space and temperature related problems. Greif et al. [1] obtained an exact solution for the problem of laminar convective flow in a vertical heated channel in the optically thin limit. They concluded that in the optically thin limit, the fluid does not absorb its own emitted radiation which means that there is no self-absorption but the fluid does absorb radiation emitted by the boundaries. Viskanta [2] investigated the forced convective flow in a horizontal channel permeated by uniform vertical magnetic field taking radiation into account. In his work, he studied the effects of magnetic field and radiation on the temperature distribution and the rate of heat transfer in the flow and found that the effect of magnetic field is to decrease fluid velocity. Later Gupta and Gupta [3] studied the effect of radiation on the combined free and forced convection of an electrically conducting fluid flowing inside an open-ended vertical channel in the presence of a uniform transverse magnetic field for the case of optically thin limit. They found that radiation tends to increase the rate of heat transfer of the fluid there by reducing the effect of natural convection.

Later, Hossain and Takhar [4] analyzed the effect of radiation using the Rosseland diffusion approximation which leads to non-similar solution for the forced and free convection of an optically dense viscous incompressible fluid past a heated vertical plate with uniform free stream and uniform surface temperature, while Hossain et al. [5] studied the effect of radiation on free convection from a porous vertical plates.

The role of thermal radiation is of major importance in the design of many advanced energy convection systems operating at high temperature and due to increase in science and technology, radiative heat transfer becomes very important in nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles and space vehicles [6–10].

In cylindrical geometry, the studies of heat generation and thermal radiation have been studied by several authors. Chamkha [11] analyzed the heat and mass transfer of a MHD flow over a moving

E-mail address: Michaeloni29@yahoo.com
Peer review under responsibility of Karabuk University.

http://dx.doi.org/10.1016/j.jestch.2016.12.009
2215-0986/© 2017 Karabuk University. Publishing services by Elsevier B.V.

Nomenclature

a	radius of inner cylinder
b	radius of outer cylinder
Da	Darcy number
g	gravitational acceleration
Gr	Grashof number
h	convective heat transfer coefficient
H	dimensionless heat generating (source) parameter
I_n	modified Bessel's function of first kind on order n. Where $n = 0, 1, 2, 3, \ldots$
J_n	Bessel's function of first kind on order n. Where $n = 0, 1, 2, 3, \ldots$
K	permeability
K_n	modified Bessel's function of second kind on order n. Where $n = 0, 1, 2, 3, \ldots$
N	radiation parameter
Nu_0	rate of heat transfer at the outer surface of the inner cylinder
Nu_1	rate of heat transfer at the inner surface of the outer cylinder
p	dimensional pressure difference
P	dimensionless pressure difference
Pr	Prandtl number
Q_0	dimensional heat generating (source) parameter
Re	Reynold number
r	dimensional coordinate
R	dimensionless coordinate
T	dimensional temperature of the fluid
T_0	initial temperature
T_w	final temperature
u	dimensional velocity of fluid
U	dimensionless velocity of fluid
Y_n	Bessel's function of second kind on order n. Where $n = 0, 1, 2, 3, \ldots$
z	dimensional coordinate
Z	dimensionless coordinate

Greek alphabets

α	thermal diffusivity
β	coefficient of thermal expansion
κ	thermal conductivity
κ^*	mean absorption coefficient
λ	aspect ratio (b/α)
σ	Stefan-Boltzmann constant
ν	kinematic viscosity
ρ	density
θ	dimensionless temperature of fluid
μ	dynamics viscosity
γ	viscosity ratio

Subscript

1	outer surface of inner cylinder
λ	inner surface of outer cylinder

permeable cylinder with heat generation or absorption and chemical reaction. He concluded that the role of Hartmann number is to decrease fluid velocity. Trujillo et al. [12] studied the heat and mass transfer process during the evaporation of water from a circular cylinder through CFD modeling. In other related work, Mujtaba and Chamkha [13] discussed the heat and mass transfer from a permeable cylinder in a porous medium with magnetic field and heat generation/absorption. Ganesan and Loganathan [14,15] investigated an unsteady natural convective flow past semi-infinite vertical cylinder with heat and mass transfer under different physical situations. Hossain et al. [16] reported the radiation conduction interaction on mixed convection from a horizontal circular cylinder using an implicit finite-difference scheme.

Also, Ganesan and Loganathan [17] studied the radiation and mass transfer effects on flow of an incompressible viscous fluid past a moving vertical cylinder. Gnaneswar and Reddy [18,19] analyzed the radiation and mass transfer effects on an unsteady MHD free convection flow of an incompressible viscous fluid past a moving vertical cylinder. Radiation effects on hydromagnetic free convective and mass transfer flow of a gas past a circular cylinder with uniform heat and mass flux was studied by Hakiem [20]. He also found that magnetic field parameter retards fluid velocity. Yih [21] analyzed the radiation effect on natural convection over a vertical cylinder embedded in a porous media. Suneetha and Bhaskar [22] analyzed the radiation and mass transfer effects on MHD free Convection flow past a moving vertical cylinder embedded in a porous medium. Other related articles on radiation effect on heat transfer of mixed convection flow for different fluid can be seen in [23–25]

The purpose of this paper is to examine theoretically the effects of thermal radiation and porosity on viscous, incompressible and heat generating fluid in a vertical annulus filled with porous material. Exact solution for temperature, velocity, skin-friction and Nusselt number are obtained and the effects of governing parameters are discussed with the aid of line graphs.

2. Mathematical analysis

Consider the steady laminar mixed convection flow of a viscous incompressible heat generating fluid. The axis of cylinder is taken along the z-axis, while r-axis is taken in the radial direction. The inner surface of the outer cylinder is assumed to be heated to a temperature T_w greater than that of surrounding fluid and outer surface of the inner cylinder having temperature T_0. The radius of the inner and outer cylinder walls are a and b respectively as

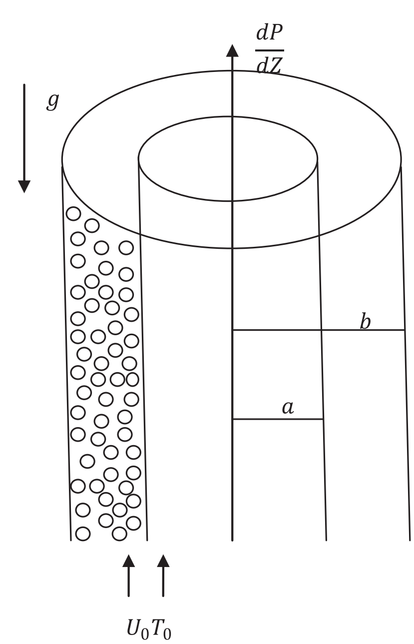

Fig. 1. Schematic diagram.