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Isotropy analysis of spherical mechanisms using an instantaneous-pole based method

Soheil Zarkandi

Department of Mechanical Engineering, Babol Noshirvani University of Technology, Babol, Iran

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A B S T R A C T

As the instant centers in planar mechanisms, the instantaneous poles can be used for instantaneous kinematic analysis of spherical mechanisms. In this paper, a novel geometrical method is presented for isotropy analysis of spherical mechanisms via the concept of instantaneous poles. First, a different form of the Jacobian matrix is formulated for multi-degree-of-freedom (multi-DOF) spherical mechanisms, based on the instantaneous poles. Then, using the obtained Jacobian matrix, isotropy analysis of spherical mechanisms is carried out, and general conditions to have isotropic configurations are determined. The proposed method is fast and applicable for all types of spherical mechanisms. At last, two illustrative examples are presented to show the efficiency of the method.

1. Introduction

A mechanism is called *isotropic* if it has at least one isotropic configuration in its workspace [1]. Moreover, a mechanism is called *fully-isotropic* if it is isotropic in its entire workspace [2]. In an isotropic configuration, the sensitivity of a mechanism is minimal with regard to both velocity and force errors and the mechanism can be controlled equally well in all directions [3]. For this reason, many researchers have addressed the isotropy analysis (i.e. determining isotropic configurations) and isotropic design of mechanisms [1–14].

Traditionally, isotropy analysis of mechanisms is carried out by the condition number of the Jacobian matrix [1–12], which is first used by Salisbury and Craig [5]. In an isotropic configuration, the condition number reaches the minimum value of unity; that is to say, the rows of the Jacobian matrix becomes mutually orthogonal and of equal Euclidean norms [6]. Zanganeh and Angeles [1] used a special structure of the forward and the inverse Jacobian matrices to define a set of conditions under which a parallel mechanism can be rendered isotropic. Carricato and Parenti-Castelli [2] presented the topological synthesis of a family of singularity-free fully-isotropic translational parallel mechanisms. Tsai and Huang [3] developed 6-DOF isotropic parallel mechanisms by a devise called isotropy generator. Klein and Miklos [4] demonstrated design techniques using positional, orientational, or spatial isotropy and presented some algorithms for locating isotropic designs without explicit evaluation of singular values. Gosselin and Lavoie [7], and Mohammadi Daniali et al. [8] used the kinematic isotropy as a design criterion for isotropic design of a class of 3-DOF planar and spherical parallel mechanisms. Qu et al. [12] determined isotropic configurations of limited-DOF parallel mechanisms based on the terminal constraints system and reciprocal screw theory. An advantage of Qu et al.'s method [12] is that there is no need to construct the general Jacobian matrix, which is a difficult process for some complex structural parallel mechanisms; however, for obtaining the terminal constraints system of the parallel mechanisms, it is required to analyze the reciprocal screws of each leg applied to the moving platform which is relatively a difficult task. Moreover, Gogu [13,14] synthesized two families of fully-isotropic parallel mechanisms with translational and Schönflies motions via theory of linear transformations and evolutionary morphology.

As the instant centers which have been extensively used in kinematic analysis of planar mechanisms (see for instance [15–19]), the instantaneous poles can be exploited for kinematic analysis of spherical mechanisms [20–24]. Recently, Di Gregorio [24] has used the novel concept of the instantaneous poles for singularity analysis of multi-DOF spherical mechanisms. He obtained a general expression of the input–output relationship of multi-DOF spherical mechanisms by exploiting the properties of the instantaneous poles and the superposition principle. In particular, his proposed expression contains only the position vectors of instantaneous poles of the single-DOF spherical mechanisms that are generated from the multi-DOF mechanism by locking all the inputs but one.

E-mail address: zarkandi@gmail.com
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In this paper, by an extension of Di Gregorio's method [24], a different form of the Jacobian matrix of a multi-DOF spherical mechanism is defined based on the instantaneous poles of the mechanism. Then, the new Jacobian matrix is used for isotropy analysis of spherical mechanisms.

2. Spherical motion and instantaneous poles

Spherical mechanisms are the mechanisms where a point of the frame, named spherical-motion center, can be considered embedded in all the links (i.e. all the mobile links are constrained to perform spherical-motions whose center is the spherical-motion center).

Kinematically, the moving links of a spherical mechanism can be considered as the co-spherically moving shells that move on the surface of a reference sphere. Without loss of generality, the radius of the reference sphere is taken as unity and the sphere is called unit sphere [20].

For two co-spherically moving shells or links (e.g. links *i* and *j* in Fig. 1), there exist two instantaneously coincident points, each belonging to the respective shell or its extension, the linear velocities of which are identical. The place of these common points is called *instantaneous pole*, henceforth referred to as *instant pole*, of the two shells and will be denoted as p_{ij} [20]. Moreover, $\mathbf{p}_{ij}$ will indicate the position vector of the instant pole p_{ij} with respect to the reference coordinate frame $O{-}xyz$ attached to the center of unit sphere (see Fig. 1). In fact, for two shells moving about a common sphere center, there are two instant poles located diametrically opposite to each other on the reference sphere, and the two shells are said to rotate instantly relative to each other about an *instantaneous pole axis* [21] that passes through two instant poles and the sphere center (Fig. 1).

Since the instant poles' positions are sufficient to fully describe the first-order kinematics of spherical mechanisms, the first-order kinematics of spherical mechanisms can be studied using only one (either positive or negative) shell of the unit sphere. Hereafter, the positive shell will be used. The points of the positive shell are defined, with respect to a reference coordinate frame $O{-}xyz$, as follows [21]

$$\begin{aligned}\text{Positive shell}: &\{(x,y,z)|x^2+y^2+z^2=1, x>0\}\\ &\cup\{(x,y,z)|z^2+y^2=1, x=0, y>0\}\\ &\cup\{(x,y,z)|x=y=0, z=1\}\end{aligned}$$

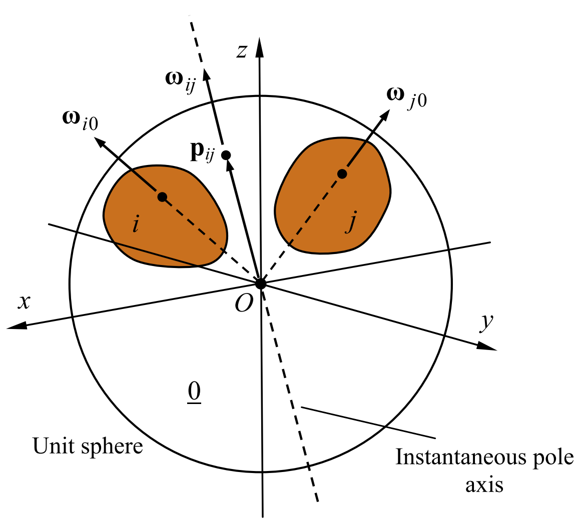

Fig. 1. The relative instant pole of two co-spherically moving links, *i* and *j*, rotating about the point *O* with angular velocities $\boldsymbol{\omega}_{i0}$ and $\boldsymbol{\omega}_{j0}$, respectively.

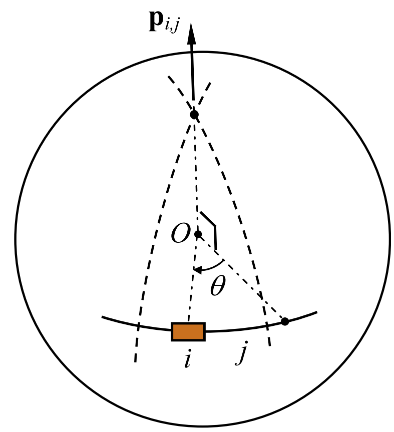

Fig. 2. A sliding joint and the corresponding instant pole.

In contrast to planar motion, there exists only one type of motion between two moving links in a spherical mechanism which is rotational. If the two links are connected to each other through a revolute joint or a rolling contact then instant pole of the links is trivial; on the other hand, If the two links are connected to each other through a sliding joint, then instant pole of the links is located on the intersection of the great circles, which are normal to the common spherical curve, the links move on it (Fig. 2). Therefore, a sliding joint can also be considered as a revolute joint.

The Aronhold-Kennedy (A–K) theorem can be stated for spherical mechanisms as follows [20]:

The relative instant poles of three co-spherically moving links lie on a unique great circle.

For detailed explanation of the instant poles and spherical motion, the reader is referred to Refs. [20,21].

3. Jacobian matrix of multi-DOF spherical mechanisms

The general input–output instantaneous relationship of a single-DOF spherical mechanism [22] involves six relative motions around the associated instant poles axes of four links: input link, '*i*', output link, '*o*', reference link, '*r*', used to evaluate the rate of the input variable $\theta_{i,r}$ and reference link, '*s*', used to evaluate the rate of the output variable $\phi_{o,s}$. The input (output) variable could be visualized as an angle between the planes containing two suitable oriented great circle arcs fixed to links '*i*' ('*o*') and '*r*' ('*s*'), respectively (see Fig. 3a). The input–output instantaneous relationship is defined as [22]

$$a\dot{\theta}_{i,r} = b\omega_{o,s} \tag{1}$$

where the signed magnitude $\dot{\theta}_{i,r}$ ($\omega_{o,s}$) is the rate of input (output) variable $\theta_{i,r}$ ($\phi_{o,s}$). Moreover, coefficients *a* and *b* depend on the configuration of the mechanism and are deduced as follows

$$a = \mathbf{p}_{r,s}\cdot(\mathbf{p}_{i,r}\times\mathbf{p}_{i,o}) \tag{2a}$$

$$b = \mathbf{p}_{r,s}\cdot(\mathbf{p}_{o,s}\times\mathbf{p}_{i,o}) \tag{2b}$$

If the reference link used to evaluate the rate of input and output variables are the same, e.g. link *r*, then a more meaningful input–output relationship can be obtained as follows [22]

$$a\dot{\theta}_{i,r} = b\omega_{o,r} \tag{3}$$

where

$$a = \mathbf{p}_{i,r}\cdot(\mathbf{p}_{o,t}\times\mathbf{p}_{i,t}) \tag{4a}$$

$$b = \mathbf{p}_{o,r}\cdot(\mathbf{p}_{o,t}\times\mathbf{p}_{i,t}) \tag{4b}$$