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Influence of magnetohydrodynamics on metachronal wave of particle-fluid suspension due to cilia motion

M.M. Bhatti [a,*], A. Zeeshan [b], M.M. Rashidi [c,d]

[a] Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, China
[b] Department of Mathematics, International Islamic University Islamabad, Pakistan
[c] Shanghai Key Lab of Vehicle Aerodynamics and Vehicle Thermal Management Systems, Tongji University, Shanghai 201804, China
[d] ENN-Tongji Clean Energy Institute of advanced studies, Shanghai 200072, China

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ABSTRACT

In this article, the influence of magnetohydrodynamics (MHD) on cilia motion of particle–fluid suspension through a porous planar channel has been investigated. The governing equations of Casson fluid model for fluid phase and particulate phase are solved by taking the assumption of long wavelength and neglecting the inertial forces due to laminar flow. The solutions for the resulting differential equations have been obtained analytically and a close form of solutions is presented. The expression for pressure rise along the whole length of the channel is evaluated numerically. The influences of all the physical parameters are demonstrated graphically. Trapping mechanism has also been discussed with the help of streamlines. It is observed that due to the influence of magnetohydrodynamics and particle volume fraction, velocity of the fluid decreases. It is also found that pressure rise shows similar behaviour for particle volume fraction and Casson fluid parameter.

1. Introduction

The term cilia is often used for "eukaryotic cells," and it is derived from the word "eyelashes," The term cilia is used when different types of cilia elements are appendages on a single cell. The inner layer of these cells is described by a cylindrical core which is known as axoneme. The axoneme contains cylindrical arrangements of molecular motors/dynein and elastic elements (known as microtubules). All the cilia elements on the surface of microorganism describe the same beating pattern. The length range of each cilium element is about 2 μm to mm and its diameter is about 0.2 μm. The shape of the cilia elements is very much similar to hair like motile appendages which can be observed in the nervous system, digestive system, male and female reproductive system. Cilia motion plays a vital role in various physiological processes such as reproduction, alimentation, locomotion and respiration. Metachronal wave that is generated due to the waves of ciliary elements propagates along out of phase direction in a distensible tube/channel. The main benefit of metachronal wave is to control the continuity of the flow and also to help us increase the amount of fluid propelled. Ciliary moments are found in different shapes depending on the ciliary system, such as oscillatory, excitable, helical, beating and planar. When the metachronal wave propagates along the same path as the effective stroke, then this phenomenon is known as symplectic metachronism, whereas when they propagate along the opposite direction, then this phenomenon is known as antiplectic. Several authors investigated analytically and numerically the motion of cilia in different situations with various biological fluids [1–5].

On the other hand, magnetohydrodynamics is very helpful to control the flow of fluid. Magnetohydrodynamics (MHD) deals with the study of electrically conducting fluids such as salt water or electrolytes, plasmas and liquid metals. The application of magnetic field can be found in various engineering process and geophysical studies. Magnetohydrodynamics is also applicable in magnetic drug targeting for different types of cancer diseases. Magnetohydrodynamics is useful and applicable in various microchannel design for creating continues and non-pulsating flow. In biomedical engineering, magnetohydrodynamics helps in the regulation of hyperthermia, MRI and magneto-fluid rotary blood pumps, etc. Magnetohydrodynamics oscillation and waves are very famous tools for astrophysical plasmas and remote diagnostic. Magnetohydrodynamics sensors are often used to measure the angular velocity. Akbar and Khan [6–8] analysed the effects of magnetohydrodynamics on metachronal beating of cilia for Casson fluid model. Ellahi et al. [9] studied the effects of magnetohydrodynamics on peristaltic flow of Jeffery fluid in a rectangular duct through a porous medium. Mekheimer et al. [10] examined the particle fluid suspension of peristaltic flow in a non-uniform annulus. Akbar and Butt [11] analysed

* Corresponding author. Tel.: +8613162146836.
E-mail address: muhammad09@shu.edu.cn (M.M. Bhatti).
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the effects of heat transfer on metachronal of wave cilia for Rabinowitsch fluid model. Later, Akbar and Khan [6–8] characterize the effects of heat transfer for bi-viscous ciliary motion fluid. Some relevant studies on the present analysis can be found from the list of references [6–8,12–29].

With the above analysis in mind, the purpose of this present investigation is to analyse the effects of magnetohydrodynamics on metachronal wave of particle–fluid suspension due to cilia motion in the uniform porous planar channel. The governing equations of particle–fluid suspension have been solved for Casson fluid model under the assumption of long wavelength and creeping flow regime. The solution for the resulting equation has been obtained analytically and a closed-form solution is presented for fluid phase and particulate phase. The impact of all the physical parameters is discussed and plotted. This paper is summarized as follows; Sec. (2) describes the mathematical formulation of the problem, Sec. (3) is devoted to methodology and solution of the problem and finally, Sec. (4) characterizes the numerical results and discussion.

2. Mathematical formulation

Let us consider the unsteady irrotational, hydromagnetic particle–fluid suspension model, which is incompressible, and electrically conducting by an external magnetic field is applied through a two-dimensional porous planar channel. A metachronal wave is travelling with a constant velocity $\tilde{c}$ that is generated due to collective beating of cilia along the walls of the channel whose inner surfaces are ciliated. We have selected a Cartesian coordinate system for the channel in such a way that is $\tilde{X}$ – **axis** taken along the axial direction and $\tilde{Y}$ – **axis** is taken along the transverse direction (See Fig. 1).

The envelop for cilia tips is assumed to be written as [6–8]

$$\bar{Y}=\bar{F}\left(\tilde{X},\tilde{t}\right)=\tilde{a}+\tilde{a}\epsilon\cos\frac{2\pi}{\lambda}\left(\tilde{X}-\tilde{c}\tilde{t}\right), \tag{1}$$

$$\bar{X}=\bar{G}\left(\tilde{X},\tilde{t}\right)=\tilde{X}_0+\tilde{a}\epsilon\alpha\cos\frac{2\pi}{\lambda}\left(\tilde{X}-\tilde{c}\tilde{t}\right). \tag{2}$$

The vertical and horizontal velocities for cilia motion can be written as [26])

$$\tilde{U}=\frac{-\dfrac{2\pi}{\lambda}\tilde{a}\epsilon\alpha\tilde{c}\cos\dfrac{2\pi}{\lambda}\left(\tilde{X}-\tilde{c}\tilde{t}\right)}{1-\dfrac{2\pi}{\lambda}\tilde{a}\epsilon\alpha\tilde{c}\cos\dfrac{2\pi}{\lambda}\left(\tilde{X}-\tilde{c}\tilde{t}\right)}, \tag{3}$$

$$\tilde{V}=\frac{-\dfrac{2\pi}{\lambda}\tilde{a}\epsilon\alpha\tilde{c}\sin\dfrac{2\pi}{\lambda}\left(\tilde{X}-\tilde{c}\tilde{t}\right)}{1-\dfrac{2\pi}{\lambda}\tilde{a}\epsilon\alpha\tilde{c}\sin\dfrac{2\pi}{\lambda}\left(\tilde{X}-\tilde{c}\tilde{t}\right)}. \tag{4}$$

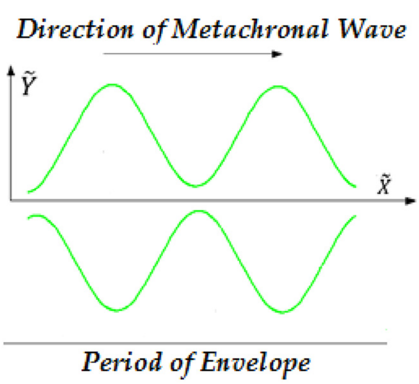

Fig. 1. Geometry of the problem.

The governing equation of motion, continuity for fluid phase and particulate phase can be written as

2.1. Fluid phase

$$\frac{\partial\tilde{U}_f}{\partial\tilde{X}}+\frac{\partial\tilde{V}_f}{\partial\tilde{Y}}=0, \tag{5}$$

$$(1-C)\rho_f\left(\frac{\partial\tilde{U}_f}{\partial\tilde{t}}+\tilde{U}_f\frac{\partial\tilde{U}_f}{\partial\tilde{X}}+\tilde{V}_f\frac{\partial\tilde{U}_f}{\partial\tilde{Y}}\right)=-(1-C)\frac{\partial\tilde{P}}{\partial\tilde{X}}+(1-C)\left(\frac{\partial}{\partial\tilde{X}}\tau_{\tilde{X}\tilde{X}}+\frac{\partial}{\partial\tilde{Y}}\tau_{\tilde{X}\tilde{Y}}\right)+CS\left(\tilde{U}_p-\tilde{U}_f\right)+J_x\times B-\frac{\mu_s}{\tilde{k}}\tilde{U}_f, \tag{6}$$

$$(1-C)\rho_f\left(\frac{\partial\tilde{V}_f}{\partial\tilde{t}}+\tilde{U}_f\frac{\partial\tilde{V}_f}{\partial\tilde{X}}+\tilde{V}_f\frac{\partial\tilde{V}_f}{\partial\tilde{Y}}\right)=-(1-C)\frac{\partial\tilde{P}}{\partial\tilde{Y}}+(1-C)\left(\frac{\partial}{\partial\tilde{X}}\tau_{\tilde{Y}\tilde{X}}+\frac{\partial}{\partial\tilde{Y}}\tau_{\tilde{Y}\tilde{Y}}\right)+CS\left(\tilde{V}_p-\tilde{V}_f\right)+J_y\times B-\frac{\mu_s}{\tilde{k}}\tilde{V}_f, \tag{7}$$

2.2. Particulate phase

$$\frac{\partial\tilde{U}_p}{\partial\tilde{X}}+\frac{\partial\tilde{V}_p}{\partial\tilde{Y}}=0, \tag{8}$$

$$C\rho_p\left(\frac{\partial\tilde{U}_p}{\partial\tilde{t}}+\tilde{U}_p\frac{\partial\tilde{U}_p}{\partial\tilde{X}}+\tilde{V}_p\frac{\partial\tilde{U}_p}{\partial\tilde{Y}}\right)=-C\frac{\partial\tilde{P}}{\partial\tilde{X}}+CS\left(\tilde{U}_f-\tilde{U}_p\right), \tag{9}$$

$$C\rho_p\left(\frac{\partial\tilde{V}_p}{\partial\tilde{t}}+\tilde{U}_p\frac{\partial\tilde{V}_p}{\partial\tilde{X}}+\tilde{V}_p\frac{\partial\tilde{V}_p}{\partial\tilde{Y}}\right)=-C\frac{\partial\tilde{P}}{\partial\tilde{Y}}+CS\left(\tilde{V}_f-\tilde{V}_p\right). \tag{10}$$

The mathematical expression for the drag coefficient and the empirical relation for the viscosity of the suspension can be described as

$$S=\frac{9\mu_0}{2\tilde{a}^2}\tilde{\lambda}(C),\quad \tilde{\lambda}(C)=\frac{4+3\sqrt{8C-3C^2}+3C}{(2-3C)^2},\quad \mu_s=\frac{\mu_0}{1-\chi C},$$
$$\chi=0.07e^{\left[2.49C+\frac{1107}{T}e^{-1.69C}\right]}. \tag{11}$$

The stress tensor of Casson fluid is defined as

$$\tau^{1/n}=\tau_0^{1/n}+\mu_s\dot{\gamma}^{1/n}, \tag{12}$$

$$\tau_{i,j}=2E_{i,j}\left(\mu_b+\sqrt{2\pi_D}/P_y\right). \tag{13}$$

In the above equation, $\pi=E_{i,j}$ we have consider $P_y=0$ Now, it is convenient to define the transformation variable from fixed frame to wave frame

$$\tilde{x}=\tilde{X}-\tilde{c}\tilde{t},\quad \tilde{y}=\tilde{Y},\quad \tilde{u}_{f,p}=\tilde{U}_{f,p}-\tilde{c},\quad \tilde{v}_{f,p}=\tilde{V}_{f,p},\quad \tilde{p}=\tilde{P}. \tag{14}$$

Introducing the following non-dimensional quantities

$$\tilde{x}=\frac{x}{\lambda},\quad \tilde{y}=\frac{y}{\tilde{a}},\quad \tilde{u}_{f,p}=\frac{u_{f,p}}{\tilde{c}},\quad \tilde{v}_{f,p}=\frac{v_{f,p}}{\tilde{c}\delta},\quad p=\frac{\tilde{a}^2}{\lambda\tilde{c}\mu_s}\tilde{p},\quad \mathrm{Re}=\frac{\rho\tilde{a}\tilde{c}}{\mu_s},$$
$$N=\frac{S\tilde{a}^2}{\mu_s},\quad M=\sqrt{\frac{B_0^2\tilde{a}^2\sigma}{\mu_s}},\quad k=\frac{\tilde{k}}{\mu_s},\quad \zeta=\frac{\mu_b\sqrt{2\pi_D}}{P_y},\quad \beta=\frac{\tilde{a}}{\lambda}. \tag{15}$$

Using Eq. (14) and Eq. (15) and taking the approximation of long wavelength and neglecting the inertial forces, then Eq. (5) to Eq. (10)

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