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Full Length Article

# Transient free convective flow in an annular porous medium: A semi-analytical approach

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ABSTRACT

The transient fully developed free-convective flow of viscous incompressible fluid between two concentric vertical cylinders filled with porous material and saturated with the same fluid have been analysed due to isothermal or isoflux heating of the outer surface of inner cylinder. The governing partial differential equations of motion and energy are transformed into ordinary differential equations using Laplace transform technique. The ordinary differential equations are then solved analytically in Laplace domain. The Riemann-sum approximation method is used to invert the Laplace domain into time domain. The solution obtained is validated by comparison with closed form solution obtained for steady states which has been obtained separately. An excellent agreement was found for transient and steady state solution at large value of time. The governing partial differential equations are also solved by implicit finite difference method to verify the present proposed method. The variation of temperature, velocity, skin-frictions and mass flow rate with dimensionless parameter controlling the present physical situation are illustrated graphically and discussed. It is observed that velocity and temperature increases with time and finally attains its steady state status. Furthermore, both velocity as well as temperature of the fluid is higher in case of isothermal heating of the outer surface of the inner cylinder compared with the isoflux heating case when the gap between the cylinder is less or equal to the radius of inner cylinder while reversed trend is observed when the gap between the cylinders is greater than the radius of inner cylinder for all considered values of time.



## 1. Introduction

Fluid flow as a result of buoyancy force due to temperature difference in a porous medium are found in wide range of application in engineering such as operation of solar collectors, cooling system in electronics equipment, water purification, ground water studies. A good survey of the existing literature shows that flow through a porous medium has been on the increase in recent years. To mention but few, Vafai and Tien [1] presented the importance of Brinkman and Forchheimer terms in forced convection over a flat plate. They obtained the resulting error in heat transfer coefficient when the viscous and inertia terms are neglected. Jha [2] extensively did the analysis of the closed-form solution of natural convection flow through a vertical annular duct filled with porous media for more general thermal boundary condition at the outer surface of the inner cylinder using the non-Darcian flow model derived by Vafai and Tien [1]. Paul and Singh [3] investigated fully developed natural convection flow between coaxial vertical cylinders partially filled with a porous material. In which they observed that the velocity is influenced by the shear stress jump condition at the interface.

In another article, Joshi [4] studied the fully developed free-convection flows in vertical annulus with two isothermal boundaries. In a related work, Singh and Singh [5] considered the effect of induced magnetic field on natural convection flow in vertical concentric annulus. In their work they found that the velocity fields are the same when the outer surface of the inner cylinder is either heated isothermal or constant heat flux when the annular gap is 1.71.

The transient free-convective flow through a vertical porous annulus has been discussed by Jha et al. [6] when the thermal boundary condition is maintained at a constant temperature or at a constant heat flux. They obtained that the steady state temperature and velocity are independent of Prandtl number in the absence of suction/injection. Javaherdeh et al. [7] studied natural

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**Nomenclature**

| | |
|---|---|
| $t'$ | dimensional time |
| $r'$ | dimensional radial coordinate |
| $U'$ | axial velocity |
| $U$ | dimensionless axial velocity |
| $R$ | dimensionless radial coordinate |
| $T_0$ | initial temperature ($t' \leqslant 0$) |
| $T_w$ | temperature of the outer surface of the inner cylinder in the case of isothermal heating ($t' > 0$) |
| $\theta$ | dimensionless temperature |
| $a$ | radius of the inner cylinder |
| $b$ | radius of the outer cylinder |
| $g$ | gravitational acceleration |
| $Q$ | dimensionless mass flow rate |
| $Pr$ | Prandtl number |
| $C_p$ | specific heat at constant pressure |
| $t$ | dimensionless time |
| $K$ | permeability of the Porous medium |
| $k$ | thermal conductivity of the fluid |
| $Da$ | Darcy number |
| $q'$ | constant heat flux |

*Greek letters*

| | |
|---|---|
| $\upsilon$ | kinematic viscosity of fluid |
| $\upsilon_{eff}$ | effective kinematic viscosity of fluid |
| $\tau$ | skin friction |
| $\rho$ | density |
| $\lambda$ | radius ratio ($b/a$) |
| $q'$ | constant heat flux |
| $\Gamma$ | ratio of viscosity |

convection heat and mass transfer in MHD fluid flow past a moving vertical plate with variable surface temperature and concentration in a porous medium.

In another related work Jha and Odengle [8] did a numerical investigation on unsteady Couette flow in a composite channel partially filled with porous material using a semi- analytical approach. This numerical scheme has been found to be in agreement with the steady state solution and it has been adopted in this research. Recently, Vanita and Kumar [9] examined the effect of radial magnetic field on natural convection flow in alternate conducting vertical concentric annuli with ramped temperature.

In the Recent past, Arpino et al. [10] presented a numerical result of transient thermal analysis of natural convection in porous and partially porous cavities. They applied the stabilized AC–CBS algorithm and are of the opinion that, the AC–CBS algorithm represents a powerful tool for the study of transient natural convection in partly porous tall cavities. Similar numerical analysis was also carried out by Massarotti et al. [11] in partially porous annuli.

Other previous works in porous medium in an annular geometry also reviewed are [12–18].

This work is aimed at providing the semi-analytical solution to the transient free-convective flow in a vertical annulus filled with porous material when thermal boundary condition at the outer surface of inner cylinder is of mixed kind while inner surface of outer cylinder is of first-kind.

## 2. Mathematical analysis

The geometry of the system under consideration for the present physical situation is shown in Fig. 1. Consider the transient fully developed free-convective flow of viscous, incompressible fluid in a vertical annulus filled with isotropic porous material. A cylindrical co-ordinate system is chosen such that the $X'$-axis is taken along the axis of the cylinders in the vertical upward direction and $r'$-axis is in the radial direction. The radii of the inner and outer cylinders are $a$ and $b$ respectively. At time $t' \leqslant 0$, both fluid and cylinders are assumed to be at temperature $T_0$. At time $t' > 0$, the temperature of the outer surface of the inner cylinder is raised to $T_w$ ($T_w > T_0$) or heat is supplied at constant rate $q'$ while the inner surface of the outer cylinder is maintained at $T_0$, causing the transient free-convection current. Since the flow is fully developed and the cylinders are of infinite length, the flow depends only on radial co-ordinate ($r'$) and time ($t'$). Under the usual Boussinesq approximation, the mathematical model representing the present physical situation in dimensional form are:

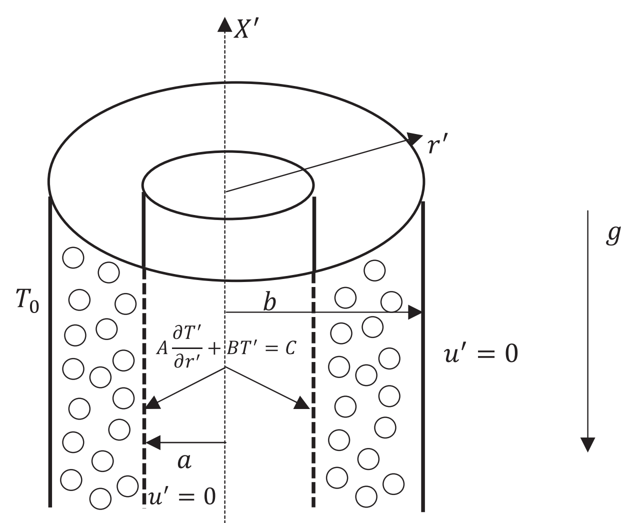


**Fig. 1.** Schematic diagram of the problem.

$$\frac{\partial U'}{\partial t'} = \upsilon_{eff}\left[\frac{\partial^2 U'}{\partial r'^2} + \frac{1}{r'}\frac{\partial U'}{\partial r'}\right] + g\beta(T' - T_0) - \frac{\nu}{K}U' \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\left[\frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'}\frac{\partial T'}{\partial r'}\right] \tag{2}$$

The initial and boundary condition relevant to the present problem under consideration are

$$\begin{aligned} &t' \leqslant 0 \quad U' = 0,\ T' = T_0 \quad a \leqslant r' \leqslant b \\ &t' > 0 \quad \begin{cases} U' = 0 & \frac{\partial T'}{\partial r'} = -\frac{q'}{k} \text{ or } T' = T_w \text{ at } r' = a \\ U' = 0 & T' = T_0 \text{ at } r' = b \end{cases} \end{aligned} \tag{3}$$

Introducing the following non-dimensional quantities:

$$t = \frac{t'\nu}{a^2}, R = \frac{r'}{a}, \lambda = \frac{b}{a}, Da = \frac{K}{a^2}, \theta = \frac{(T' - T_0)}{\Delta T},$$
$$Pr = \frac{\boldsymbol{\mu c_p}}{\mathbf{k}}, U = \frac{U'}{U_0}, U_0 = \frac{g\beta a^2 \Delta T}{\nu} \tag{4}$$

where $\Delta T = T_w - T_0$ or $\frac{q'a}{k}$ according as inner cylinder is maintained at constant temperature $T_w$ or constant heat flux $q'$, respectively. Using Eq. (4), the dimensionless momentum and energy equations are