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Full Length Article

Numerical analysis of hydromagnetic micropolar fluid along a stretching sheet embedded in porous medium with non-uniform heat source and chemical reaction

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ABSTRACT

This paper presents the effects of non-uniform heat source and chemical reaction on the convected flow, heat and mass transfer of a micropolar fluid along a stretching sheet embedded in a porous medium in the presence of a volumetric non-uniform heat source. The generalization of the earlier studies centers round three aspects:

(i) The flow is made to pass through a porous medium characterized by a non-Darcian model affecting the momentum equation.
(ii) The presence of non-uniform heat source modifying the energy equation.
(iii) Consideration of chemically reactive species characterized by an order of chemical reaction modifying the equation of species concentration.

The governing equations of the flow have been transformed into ordinary differential equations by using similarity transformation technique and solved using the Runge-Kutta method associated with shooting technique. The numerical solutions are achieved showing the effects of pertinent parameters. For verification of the present findings the results of this study have been compared with the earlier works in particular cases; Darcian and non-Darcian fluids are discussed separately. It is worth reporting that effect of porosity of the medium combined with inertia gives rise to a transverse compression producing thinner boundary layer the solution by finite element method (FEM) and Runge–Kutta method, do agree within a reasonable error limit.

1. Introduction

In the last few decades, the theory of micropolar fluid has attracted significant attention among the engineering community due to the limitation associated with Newtonian fluids. The Newtonian fluid cannot characterize the flow in presence of suspended particles. On the other hand micropolar fluids could be able to model the fluid in presence of dust particle. The presence of micropolar fluid can be found in dumb-bell molecules, polymer fluids, fluids suspensions and animal blood. Also, micropolar fluids have significant practical applications to boundary layer and heat and mass transfer area. A large number of research papers have been available in the literature on the thermal boundary layer flows [1,2].

Pioneering work in micropolar fluids field has been carried out by Eringer [3], and explained that the micropolar fluid defined by the inertial characteristics of the substructure particles which go through rotation. Kim and Lee [4] studied the problem of the micropolar fluid over a semi-infinite vertical moving porous plate in the presence of magnetic field. In their study, they considered electrically conducting oscillatory two dimensional laminar viscous incompressible flows. Sakiadis [5] studied the behavior of laminar boundary layer on a moving flat surface. The obtained results showed very good agreement between two methods of solution. Grubka and Bobba [6] investigated the influence of surface temperature change on continuous and linearly starching surface. The formulated mathematical problem was solved by using a series solution to the energy equation based on Kummer's functions. It was found that the magnitude of the temperature

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Nomenclature

A^*, B^*	coefficients of space and temperature dependent heat source/sink
b	stretching parameter
B_o	magnetic field of constant strength
c_f	drag coefficient
C_f	local skin friction coefficient
c_p	specific heat
C	concentration of the fluid
C_w	stretching sheet concentration
C_∞	ambient concentration
Ec	Eckert number
F	inertia-coefficient
g	microrotation profile
j	reference length
K	material parameter
Kp	porous parameter
k_p^*	permeability of the porous medium
k_f	thermal conductivity
M	magnetic parameter
Nu_x	local Nusselt number
Pr	generalized Prandtl number
q'''	non-uniform heat source/sink
Re_x	local Reynolds number
s	surface condition parameter
Sc	Schmidt number
Kc	chemical reaction parameter
T	temperature of the fluid
T_w	stretching sheet temperature
T_∞	ambient temperature
u_w	characteristic velocity
u	velocity along the x-direction
v	velocity along the y-direction
x, y	coordinates

Greek symbols

α^*	space dependent heat source/sink parameter
β^*	temperature dependent heat source/sink parameter
$\theta(\eta)$, $\phi(\eta)$	non-dimensional temperature and concentration parameter
η	similarity variable
υ	kinematic viscosity
α	thermal diffusivity
τ	stress tensor
σ	electrical conductivity of the fluid
ρ	density of the fluid
τ_{xy}	shear stress component of the stress tensor

parameter affects the direction and quantity of the heat flow. Minkowycz et al. [7] studied the effect of surface mass transfer in a porous medium using non similarity solution.

In recent years, several studies have been conducted on convective transport phenomena in a porous medium [8] due to its large practical applications such as geothermal reservoir, petroleum industry, thermal insulation engineering, soil mechanics, blood flow and artificial analysis. Instead of porous medium the flow characteristic in presence of stretching sheet has been studied by many flow conditions. Pioneering work in this area has been performed by Crane [9], who studied 2D boundary layer flow of an incompressible and viscous fluid. The velocity of the system was varied linearly from a fixed point on the sheet. Later, McCormack and Crane [10] discussed the physical characteristics of boundary layer flow over an elastic stretching sheet, where the sheet was moving along its own plane direction. Ali [11] analyzed general effect of power law surface and temperature variation on heat transfer characteristics. Ali and Magyari [12] studied the unsteady fluid and heat flow induced by a submerged stretching surface while its steady motion is slowed down gradually by using similarity analysis.

Mukhopadhyay and Layek [13] analyzed the effect of various fluid viscosity and thermal radiation on incompressible fluid over a vertical stretching porous plate. Pal [14] studied the two dimensional incompressible stagnation point flow over a stretching sheet by considering the effect of bouncy forces and thermal radiation. Abo-Eldahab and El-Aziz [15] studied electrically conducted mixed convection boundary layer flow over a continuous inclined stretching surface. Abel et al. [16] analyzed the effects of non-uniform heat source on viscoelastic fluid flow and heat transfer over a stretching sheet, while Bataller [17] reported the effect of both viscous dissipation and thermal radiation for the same fluid flow. Pal and Mondal [18] studied the effect of variable viscosity, Ohmic dissipation and non-uniform heat and source/sink on non-Dracy magnetohydrodynamics flow through a stretching sheet embedded in a porous medium. Mabood et al. [19,20] numerically studied the magnetohydrodynamics (MHD) slip flow and heat transfer problems on stretching sheet. The effect of heat and mass transfer of an electrically conducting nanofluid over a stretching sheet has been studied by Mabood [21]. Sandeep and Sulochana [22] studied the influence of heat and mass transfer of non-uniform source/sink on micropolar fluid over a stretching/shrinking sheet. The proposed model was solved by shooting technique using MATLAB package. Pal and Biswas [23] investigate the oscillatory MHD micropolar fluid in a porous medium with chemical reaction. Recently, Mabood et al. [24] investigated the effect of heat sourec/sink and Soret on MHD convective flow considering micropolar fluid with radiation.

Bhukta et al. [25] have analyzed heat and mass transfer on MHD flow of a viscoelastic fluid through porous media over a shrinking sheet. Recently, the flow characteristic of Jeffrey fluid in the boundary layer has been investigated over stretching sheet [26,27] by using Homotopy Analysis Method (HAM). The heat and mass transfer rate of boundary layer flow of different fluid could be controlled by using magnetic field. Many researchers have been studied the influence of transverse magnetic field on Newtonian and non-Newtonian fluids over a stretching sheet owing to some specific characteristic of magnetic field. Anika et al. [28,29] numerically investigate the effect of hall and ion-slip current on boundary layer flow in presence of transverse magnetic field. Rashidi et al. [30,31] studied the unsteady MHD Newtonian flow over a rotating stretching disk using HAM, Artificial Neural Network (ANN) and Particle Swarm Optimization (PSO) algorithm. Bhattacharyya [32] analyzed the effect of first order chemical reaction in boundary layer stagnant point flow over a stretching/shrinking sheet by using shooting technique.

Kumar [33] has studied the dynamics of hydromagnetic micropolar flow along a stretching sheet considering both effect of heat and mass transfer. His work was confined to the flow within the regions i.e. velocity, thermal and concentration boundary layers. It has considered that the mass transfer of species without chemical reaction. Further, the flow without porous matrix has been considered in the study of Kumar [33]. Moreover, though he has considered the viscous dissipation and Julian dissipation effect in energy equation but not considered any volumetric heat source either constant or variable strength which are of usual occurrence.

As the literature is beset with stabilizing/destabilizing effect of heat source on the flows in boundary layer, the necessity of inclusion of heat source is warranted. Add to it, the mass transfer