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The newsvendor problem with capacitated suppliers and quantity discounts

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ABSTRACT

We consider a seller who stocks an item in anticipation of a single selling season in which demand for the item is uncertain. The seller may order stock of the item from multiple suppliers, each of which offers a quantity discount pricing structure and has production volume limits. The seller seeks to minimize its total procurement plus expected overstock and understock costs, resulting in an objective function that is neither convex nor concave in the decision variables in general. We provide an algorithmic approach that permits solving this non-convex problem in pseudopolynomial time by solving a set of 0–1 multiple choice knapsack subproblems. We also provide an efficient heuristic solution algorithm and demonstrate the algorithm's asymptotic optimality in the number of suppliers under mild assumptions on the problem data and under certain quantity discount structures. The results of a set of computational tests demonstrate the superior performance of the knapsack-based algorithms when compared with a commercial solver.

1. Introduction

The single-period newsvendor problem is perhaps the most well-studied problem in the area of inventory planning under uncertain demand. This problem considers a decision maker who stocks an item in order to meet uncertain demand during a single selling period or season. The decision maker incurs a purchasing cost associated with the amount of stock acquired, in addition to costs and/or revenues that arise when the stock level does not equal the demand observed in the selling season. If the stock level exceeds demand, any remaining stock either produces revenue via an associated salvage value, or a disposal (or holding) cost if the remaining stock cannot be salvaged. If demand exceeds the stock level, then in addition to the opportunity cost associated with lost sales, a cost associated with a loss of customer goodwill may be incurred, reflecting an attempt to quantify an impact on future lost sales. The decision maker seeks to maximize the expected profit (or, equivalently, minimize the expected cost when opportunity cost is included) associated with stocking and selling an item, which depends on the stock level. The applicability of this model to numerous contexts involving products with short life cycles, such as fashion and technology goods, has led to an extensive volume of literature on the newsvendor problem and its many generalizations and extensions.

Under the classical approach to this problem, the newsvendor seeks an optimal order quantity, Q, for the selling season. The period's demand is characterized by a random variable X with a continuously defined probability density function (pdf) f and cumulative distribution function (CDF) F. For each unit stocked, the newsvendor pays a unit cost of c to an external supplier with unlimited capacity. If demand exceeds the stock level, a penalty cost of p is incurred for each unit of demand in excess of the stock level; if the stock level exceeds demand, a unit cost of h is incurred for each unsold unit at the end of the selling period. Then, under the standard expected-cost-minimization approach, the optimal order quantity satisfies $F(Q) = (p - c)/(p + h)$.

This paper considers a practical generalization of the newsvendor problem to account for multiple interchangeable suppliers, each of which has finite supply capacity, violating the assumption of a single supplier with unlimited capacity who can supply any quantity at a cost of c per unit. Clearly, when the available supply of the item is unlimited and the inventory procurement cost is linear in the quantity stocked, the resulting problem is the classical newsvendor problem described above. In a variety of practical settings, however, supply availability and costs may violate the assumption of an unlimited supply that is linear in the quantity ordered. In particular, it is often the case that the inventory planner has multiple suppliers from which it can choose.

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Consider, for example, a product for which most of the demand occurs relatively early after its initial appearance in the market, such as a cellphone or other electronic device, which may become obsolete in a short time (as newer versions arrive to the market). Such products may have various contract producers available to manufacture the item, perhaps in different geographical areas. These suppliers are not unlikely to have different production costs, and consequently might offer different pricing terms for production. Moreover, each such supplier typically faces some limit on the amount it can supply prior to the selling period, and may offer incentives to its customers, including quantity discounts to encourage high capacity utilization. This work revisits the newsvendor problem in order to account for these practical features, including capacitated suppliers and quantity discounts. We note that Burke, Carillo, and Vakharia (2007) considered a related version of this problem without quantity discounts, focusing on the benefits of diversification through multiple suppliers, while Zhang and Zhang (2011) considered a variant with a fixed plus linear cost associated with each supplier. Zhang and Chen (2013) generalized this latter model to account for all-unit discounts and applied a Bender's decomposition algorithm to solve the resulting large-scale mixed integer nonlinear optimization problem. In this paper we consider a general class of supplier cost structures that encompasses three well-known quantity discount structures in practice (marginal, all-units, and carload discounts, which we later define more precisely) and show that this class of problems can be solved via a sequence of 0–1 multiple choice knapsack subproblems which, although not trivial, constitute a manageable problem class with respect to solution time requirements.

This paper's contributions are as follows. We provide a model for the broadly applicable newsvendor problem with multiple capacity-constrained suppliers who offer a variety of different kinds of quantity discounts. The resulting model has an objective that is non-convex, implying that the problem is not generally well behaved. Despite this, we show that the problem can be solved via the solution of a polynomial number of 0–1 multiple choice knapsack problems under a broad set of supplier pricing structures. Thus, we show that the problem can be solved in pseudopolynomial time in the worst case for several common quantity discount pricing structures. Based on this exact solution approach, we derive an extremely fast and simple heuristic solution method and demonstrate the asymptotic optimality of this heuristic as the problem size grows under concave quantity discounts. The result is a very fast solution method that provides near-optimal solutions for what appears to be a daunting problem class.

The rest of this paper is organized as follows. Section 2 next reviews related literature on the newsvendor and knapsack problems. Section 3 introduces the general case of our model, defines three different common quantity discount structures used in practice, and provides optimization problem formulations for a class of multiple choice knapsack subproblems that can be used to solve the problem under these discount structures. This section also presents a dynamic programming solution approach for these subproblems, and characterizes the worst-case complexity required to solve the class of problems we consider. In Section 4, we consider the marginal and carload quantity discount structures in greater detail, and demonstrate how the special structural properties of these problems lead to additional efficiencies in their solution. Section 5 next discusses a fast heuristic solution method based on a linear underestimation of each supplier's quantity discount structure, and shows that under certain assumptions, this heuristic solution is asymptotically optimal as the number of suppliers increases. Section 6 provides the results of a set of computational tests intended to characterize the performance of our algorithms, while Section 7 concludes.

2. Related work

This section provides a review of relevant literature on the newsvendor problem and related extensions. We also provide a brief review of the well-known class of knapsack and multiple choice knapsack problems, which, as we later discuss, will become relevant to our solution approach for the class of newsvendor problems we consider.

2.1. The newsvendor problem

The newsvendor problem has been under study for decades. The problem was first discussed mathematically in 1888 by Edgeworth (1888), who used the concepts and tradeoffs inherent in the newsvendor model in a banking-related context, using a normal distribution to set the level of cash reserves to meet customer cash demands. Morse and Kimball (1951) were the first to use the term "newsboy" in referring to an example of the problem in their book. Qin, Wang, Vakharia, Chen, and Seref (2011) and Khouja (1999) provided literature surveys for this general problem class. Work on variants of the newsvendor problem has considered various problem aspects. Some of this effort has focused on demand specifications and assumptions, including the degree of information available about demand prior to the selling season. Many other extensions to the newsvendor problem have been suggested, focusing on various problem dimensions such as objective function forms, utility functions and risk, and multi-product cases.

2.2. Order cost assumptions

One important factor in broadening the applicability of the newsvendor model lies in considering more practical and general classes of order costs. The suppliers from which a newsvendor might order often observe economies-of-scale in production and distribution operations. Consequently, quantity discounts are sometimes employed in an effort to increase total sales and profit, while increasing capacity utilization. Thus, a segment of the literature has focused on various order cost and discount structures.

Jucker and Rosenblatt (1985) considered quantity discounts from the supplier's perspective, including "all-unit quantity discounts" (when the order quantity exceeds a threshold, all units ordered are discounted), "incremental quantity discounts" (when the order quantity exceeds a threshold, only units ordered in excess of the threshold are discounted), and "carload-lot discounts" (wherein some number of units may effectively be ordered for free if ordering a full carload of capacity). They argued that the behavior of the newsvendor in the case of "all-unit quantity discounts" is closely affected by disposal costs of the excess inventory, and showed that this type of discount affects the newsvendor's behavior in more complicated ways than previous literature suggests. Pantumsinchai and Knowles (1991) considered a different structure in which the order consists of a number of standard sized containers. They proposed algorithms to find the optimal order quantity as well as the optimal number of containers.

Khouja (1996) analyzed a newsvendor problem with an emergency supply option and showed that the existence of an emergency supply option can improve the objective. Lin and Kroll (1997) considered a dual performance measure system that seeks to "maximize the expected profit subject to a constraint that the probability of achieving a target profit level is no less than a predetermined risk level," while assuming quantity discounts from the supplier. Arcelus, Kumar, and Srinivasan (2005) analyzed optimal ordering and pricing policies under the assumption that the supplier offers a discount or a rebate directed to end customers. Qin et al. (2011) analyzed three types of quantity discount structures

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