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Decision Support

Uniqueness and multiplicity of market equilibria on DC power flow networks

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ABSTRACT

We consider uniqueness and multiplicity of market equilibria in a short-run setup where traded quantities of electricity are transported through a capacitated network in which power flows have to satisfy the classical lossless DC approximation. The firms face fluctuating demand and decide on their production, which is constrained by given capacities. Today, uniqueness of such market outcomes are especially important in more complicated multilevel models for measuring market (in)efficiency. Thus, our findings are important prerequisites for such studies. We show that market equilibria are unique on tree networks under mild assumptions and we also present a priori conditions under which equilibria are unique on cycle networks. On general networks, uniqueness fails to hold and we present simple examples for which multiple equilibria exist. However, we prove different a posteriori criteria for the uniqueness of a given solution and thereby specify properties of unique solutions.

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1. Introduction

We consider a short-run model for a liberalized power market in which producers and consumers trade electricity, which is then transported through a capacitated network. In our model, power flows are modeled by the classical lossless DC approximation of AC power flows. For this setting, we study questions of uniqueness and multiplicity of market equilibria on different types of networks like trees, cycles, and general networks. As usual, the wholesale electricity market is modeled by a mixed nonlinear complementarity system that is made up of the optimality conditions of the players of our market model and additional market clearing constraints; cf, e.g., Hobbs and Helman (2004) or the book (Gabriel, Conejo, Fuller, Hobbs, & Ruiz, 2012). The players are electricity consumers with fluctuating and elastic demand, electricity producers that are constrained by given generation capacities, and the transmission system operator (TSO) who operates the network. While producers and consumers are only constrained by simple bound constraints, the network flows controlled by the TSO have to satisfy the lossless DC power flow model constraints. Thus, the TSO has to cope with loop flows, in particular. The consideration of such loop flows in power market models is of great practical importance. In Europe, the market organization is changed from capacity-based to flow-based market coupling (cf, e.g., Aguado et al., 2012; Van den Bergh, Boury, & Delarue, 2016) and thus has to deal with loop flows. Moreover, nodal pricing is current practice in parts of the US and Canada; cf, e.g., Ehrenmann and Neuhoff (2009) and Department of Energy (2017).

Our focus in this paper is on questions regarding uniqueness and multiplicity of market equilibria on DC networks. Besides being a classical topic of mathematical economics, uniqueness of market outcomes is an important question both from a theoretical and practical point of view. In today's liberalized electricity markets, different agents make decisions that are based on the market design—e.g., nodal or zonal pricing. For instance, the TSOs make investment decisions depending on the anticipation of future market outcomes or regulators adjust more specific regulations—e.g., network fees—based on the properties of the underlying regime. Uniqueness of market outcomes typically is an important precondition for reasonable analyses of complementary decisions of the mentioned agents. In addition, today's operations research literature on energy markets often integrate models similar to the one discussed in this paper into more complex multilevel frameworks in order to evaluate more complicated market models; cf, e.g., Hobbs, Metzler, and Pang (2000), Daxhelet and Smeers (2001), and Hu and Ralph (2007) and the references therein. These frameworks can, for instance, be of bi- or general multilevel type for evaluating the efficiency of specific market designs in which different players act. Moreover, the mentioned frameworks can also capture multiperiod settings in long-run models that incorporate investment

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decisions of the different players in the market. In both situations, uniqueness of the outcomes of the short-run model (e.g., used as the lower level in a bilevel model and thus considered as a parameterized optimization problem in dependence of the upper level's decisions) discussed in this paper is of particular importance for the theoretical study of the overall model as well as for the development of effective solution methods for these, typically hard, multilevel or multiperiod problems. For a detailed mathematical discussion of the importance of uniqueness of lower levels in multilevel models see, e.g., Dempe (2002); in particular Chapter 4. Multilevel models with a DC power flow model on a lower level can be found in, e.g, Hobbs et al. (2000), Daxhelet and Smeers (2001), Hu and Ralph (2007), Ruiz and Conejo (2009) or in Grimm, Martin, Schmidt, Weibelzahl, and Zöttl (2016), Grimm, Kleinert, Liers, Schmidt, and Zöttl (2017a) as well as Kleinert and Schmidt (2018), where the lower-level DC formulation also depends on upper-level network design decisions.

This paper builds on the paper (Grimm, Schewe, Schmidt, & Zöttl, 2017b) in which the authors analyze a comparable setting: On the one hand, they consider models of capacitated networks without DC power flow constraints. On the other hand, their model is a long-run model in which investment decisions of electricity producers in new generation capacity is also taken into account. In this paper, we only consider the short run but integrate a more detailed flow model into our setup. We contribute to the rich literature on liberalized power markets in general and on uniqueness questions in particular. For instance, Metzler, Hobbs, and Pang (2003) also consider power market equilibria that are constrained by a linear DC network model with arbitrage. The authors study bilateral contracts between producers and consumers in a Nash–Cournot setting. They also formulate their market model as a mixed linear complementarity problem (MLCP) and prove uniqueness of the corresponding equilibria. The mentioned paper builds on the article (Hobbs, 1999), in which an arbitrage-free Nash–Cournot model of bilateral and POOLCO markets constrained by a linear DC model is also formulated as a complementarity problem. In the latter paper, uniqueness aspects are not considered but mentioned for future work. Hobbs and Rijkers (2004) considers an oligopolistic market model with arbitrage and a linear DC network to analyze market power of generators. The resulting mixed complementarity model is further studied in Pang, Hobbs, and Day (2003), where the classical theory of linear complementarity problems (LCPs) is used to prove uniqueness; see Cottle, Pang, and Stone (2009) for a detailed presentation of this LCP theory. Ruiz and Conejo (2009) study a pool-based electricity market to determine the optimal offering strategy of a strategic power producer. The authors use a bilevel programming model in which the lower-level problem represents a welfare-maximizing market clearing with respect to a DC network model. This model is very similar to the one studied in this paper. However, uniqueness of solutions is not considered in Ruiz and Conejo (2009).

Another discussion using a model very similar to ours is given in Holmberg and Lazarczyk (2012). The authors compare nodal and zonal pricing schemes and prove uniqueness of a DC power flow based market model. However, they assume strictly convex cost functions so that uniqueness of the resulting strictly convex optimization problem follows from standard theory. Due to the effort of calibrating strictly convex models for computational studies, many authors refrain from this assumption and use linear cost functions; cf, e.g., Chao and Peck (1998), Ehrenmann and Smeers (2011), Hobbs and Pang (2007), Gabriel et al. (2012) as well as Grimm et al. (2017a, 2016), and Grimm et al. (2017b). Very recently, Bertsch, Hagspiel, and Just (2016) also considers a long-run model and study congestion management regimes in an inter-temporal equilibrium model. The authors of the latter paper discuss the importance of uniqueness of equilibria in such multiperiod models. However, a detailed analysis of this issue remains open and is only partly addressed by satisfying certain assumptions for which we show that they are not sufficient for uniqueness. This example together with our results on multiplicity of market equilibria on general networks indicates that both models and solution methods have to be chosen very carefully in this context—an issue that is also discussed in Wu, Varaiya, Spiller, and Oren (1996). In summary, market model outcomes only seem to be proven unique for the mixed LCP case with arbitrage and for the case of strictly convex cost functions. However, the latter assumption is often not satisfied in computational equilibrium models as discussed above.

The main contribution of this paper is to close this gap in the literature: namely to study uniqueness and multiplicity of market equilibria that are subject to DC power flow networks without arbitrage and not necessarily strictly convex cost functions. We show that uniqueness of market equilibria on general networks typically fails to hold by presenting simple examples with multiple solutions. Furthermore, we characterize the situations in which multiple solutions appear. We can, however, prove uniqueness in special cases: Market equilibria are unique on radial, ie, tree-like, networks where no loop flows need to be considered. This is a direct consequence of the results shown in Grimm et al. (2017b). Moreover, we derive a priori conditions on cycle networks, ie, conditions that solely rely on properties of the problem's data, under which we can prove uniqueness of equilibria. It turns out that these criteria both depend on production costs and the data of the network's lines. Finally, we prove a posteriori conditions for uniqueness on general networks. That means, the latter conditions can be used ex post to check whether a given solution is unique. Our results cover the case of perfect competition, which is a commonly used economic setting in the context of power market modeling; cf, e.g., Boucher and Smeers (2001) as well as Daxhelet and Smeers (2007). Since models of strategic behavior typically make it much harder to establish uniqueness results, cf, e.g., Zöttl (2010), we refrain from discussing the case of imperfect competition—all the more in the light of multiplicity of equilibria that we already obtain under perfect competition in the case of general networks. In comparison to Grimm et al. (2017b) the following is noteworthy: Both the market model with a simple network flow model studied in Grimm et al. (2017b) yields unique solutions and, as we will show, the physics model studied in this paper without a market model yields unique solutions. However, the combination of both yields multiple solutions.

The rest of the paper is structured as follows. In Section 2 we present our market model both as a mixed nonlinear complementarity problem and as an equivalent optimization problem that we study in the following. Section 3 contains basic known and new results that are used throughout the rest of the paper. Afterward, Section 4 proves uniqueness for tree networks and Section 5 for cycle networks. Then, in Section 6 we show that multiple equilibria arise quite naturally on general networks, derive different a posteriori uniqueness conditions for the general case, and thereby describe properties of unique solutions. The paper closes with some concluding remarks in Section 7.

2. Market equilibrium modeling

We consider electricity networks that we model by using a connected digraph $G:=(N, A)$ with node set N and arc set A. Subsequently, all player models of our overall market model are stated. Since we consider perfectly competitive markets, all players are price takers and their optimization problems are formulated using exogenously given market prices π_u at every node $u \in N$. The model is based on standard electricity market models as discussed in, e.g., Hobbs and Helman (2004) and Gabriel et al. (2012).

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