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Measuring exposure to dependence risk with random Bernstein copula scenarios

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ABSTRACT

This paper considers the problem of measuring the exposure to dependence risk carried by a portfolio with an arbitrary number of two-asset derivative contracts. We develop a worst-case risk measure computed over a set of dependence scenarios within a divergence restricted region. The set of dependence scenarios corresponds to Bernstein copulas obtained by simulating random doubly stochastic matrices. We then devise a method to compute hedging positions when a limited number of hedging instruments are available for trading. In an empirical study, we show how the proposed method can be used to reveal an exposure to dependence risk where usual sensitivity methods fail to reveal it. We also illustrate the ability of the proposed method to generate parsimonious hedging strategies in order to reduce the exposure to dependence risk of a given portfolio.

1. Introduction

Derivative contracts written on several underlying assets represent a sizeable part of banks trading portfolios. Usually written on stocks or equity indices, they are sold to asset managers and insurance companies or traded in the interbank market. To a lesser extent these derivative contracts are also written on other asset classes such as commodities and foreign exchange rates. The value of a portfolio gathering such contracts depends heavily on the dependence, or correlation, chosen to describe the joint behavior of the underlying assets. When the chosen dependence is changed, the portfolio value changes as well, leading to a profit or to a loss for the bank holding the portfolio. The eventuality of a loss in this context is what we consider to be an exposure to *dependence risk*. The two main reasons underpinning a change of the chosen dependence are an evolution of the conditions prevailing on the market and a mis-estimation/specification of the dependence. The former corresponding to market risk and the latter to model risk. Because we want to focus only on dependence risk, we need to isolate it and this is the reason why we do not consider other potential sources of profits and losses in terms of portfolio value. Isolating the exposure to dependence risk allows us to better understand it and to compute strategies specifically designed to hedge it.

For a bank holding a trading portfolio of multi-asset derivative contracts, it is a crucial question to measure its exposure to dependence risk. Another situation where this question is crucial is when a market participant is shutting down its activities in this type of derivatives and calls for bids from other banks to buy its trading portfolio. When one bids to acquire this portfolio, of course the value is a key aspect, but the amount of risk coming with it can be even more important. The amount of risk determines the attention to be devoted to the management of the portfolio once it has been acquired. The amount of risk also directly determines the capital to be set aside and that will not be available for other business lines.

An acceptable answer to this question should take into account the heterogeneity of the contracts recorded in the considered portfolio. This heterogeneity can concern the payoff of the product (option on spread, on basket, on the min or max, etc.), its features (basket weights, strike, etc.) as well as the direction of the position (long or short).

In the banking industry, the approach frequently relied upon is a partial derivatives analysis (sensitivities) with respect to the parameters of a parametric model. This approach is used at the same time to assess and hedge the exposure to dependence risk. Often, the parametric model is a bivariate Gaussian model and the sensitivity of the portfolio value is computed with respect to the correlation parameter. This approach is parametric as it reduces the dependence model to the choice of a single parameter. In turn, it may give an incomplete assessment of the risk exposure. Problems arise when one finds a very low or nonexistent exposure using this one-parameter sensitivity method, where a more detailed analysis unveils a significant exposure. This partial derivatives approach for spread options is commented in Carmona and Durrleman (2003),

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Alexander and Scourse (2004) and Venkatramanan and Alexander (2012). For a broad and recent presentation of issues related to dependence modeling, we refer to Werner, Bedford, Cooke, Hanea, and Morales-Nápoles (2017).

In this paper, we devise a method to measure the exposure to dependence risk carried by a portfolio of two-asset options in order to have an alternative to the partial derivatives approach. We keep the spirit of a local analysis around the initial dependence but we explore a set of potential changes much wider than just infinitesimal variations of a small number of parameters attached to a parametric model. The risk measure we obtain is able to handle heterogeneous portfolios in the understanding given above. It is a monetary and coherent measure of risk as described in Artzner, Delbaen, Eber, and Heath (1999) and Föllmer and Schied (2002a); 2002b). It can be directly used for computing additional capital requirements (regulatory or internal assessments). Our approach corresponds to a local analysis and the idea is not to replace quantile based measures[1] but to complement them by shedding light on different aspects of the exposure (for example stemming from model uncertainty).

The measure we introduce is formalized as a worst-case loss among scenarios. As such, it is linked to the stress-testing approach for which the regulators currently have a liking. The worst-case framework to risk measurement founds its foundations in Gilboa and Schmeidler (1989). It has been applied to the assessment of risk exposures of derivative contracts in Cont (2006) and Coqueret and Tavin (2016). With a focus on dependence modeling it is also used to quantify the uncertainty of risk measurements in Bernard, Denuit, and Vanduffel (2017). A recent study on coherent risk measures and capital requirements can be found in Balog, Bátyi, Csóka, and Pintér (2017).

In our setup, the dependence between the two underlying assets is modeled by a copula function that is a joint distribution function on the unit square with uniform marginals. Hence, choosing a dependence structure to model the underlying assets comes down to choosing an element in a set of functions. We consider the initial (prior) dependence to be known and already fitted to the available market data. The uncertainty regions we adopt to compute the proposed worst-case risk measure are based on Bregman divergences with respect to the prior dependence function. A standard example of Bregman divergence is the relative entropy (also named Kullback–Leibler divergence). In this approach, the updated dependence may lie within a fixed divergence radius with respect to the prior dependence. On this aspect we refer to Ben-Tal, den Hertog, De Waegenaere, Melenberg, and Rennen (2013), Breuer and Csiszár (2013, 2016) and Csiszár and Breuer (2015). In a similar spirit, Barrieu and Scandolo (2015) and Lux and Papapantoleon (2016) develop an analysis of model risk for risk measures.

To compute the proposed risk measure we have to explore the set of copula functions which has a nice structure but remains infinite dimensional. Instead, we explore the set of Bernstein copulas that are dense in the set of all copulas. A Bernstein copula of order m can be parametrized by a $m \times m$ doubly stochastic matrix[2] that is a square matrix with non-negative real entries of which each row and column sum to one. The set of doubly stochastic matrices is named the Birkhoff polytope. For a detailed introduction do doubly stochastic matrices and their properties we refer to Brualdi (2006). Sampling the set of $m \times m$ doubly stochastic matrices allows us to generate random Bernstein copulas. Each random Bernstein copula generated being a dependence scenario we take into account for the computation of our risk measure. The theory underlying the definition of Bernstein copulas can be found in Li, Mikusinski, Sherwood, and Taylor (1997) and Li, Mikusinski, and Taylor (1998). This family of copulas is further studied in Durrleman, Nikeghbali, and Roncalli (2000) and Sancetta and Satchell (2004). For the theory of sampling the Birkhoff polytope, we refer to Sinkhorn (1964), Smith (1984), Lovász and Simonovits (1993), Chan and Robbins (1999), Cappellini, Sommers, Bruzda, and Życzkowski (2009) and Diaconis, Lebeau, and Michel (2012).

The main contributions of this paper are the following:

- First, we propose a new measure of dependence risk for two-asset derivative portfolios based on a worst-case and scenario-based approach. The proposed measure is coherent and can be used to compute capital requirements as well as determining parsimonious hedging strategies. Two empirical applications of this method are developed.
- Second, we detail a method to generate random dependence scenarios by random sampling the set of doubly stochastic matrices used to parametrized Bernstein copulas.

The remainder of the paper is organized as follows. In Section 2, we describe our financial framework and probabilistic setting. Section 3 introduces the notion of dependence risk and discusses the approach chosen for its measurement. In Section 4, we propose a version of our risk measure whose computation relies on a set of random Bernstein copula scenarios and we comment its properties. A way to determine hedging strategies is also formalized. In Section 5, we review methods to generate random Bernstein copulas using tools from random matrix theory. Section 6 gathers two empirical applications of the proposed methods. Section 7 concludes. A companion Supplementary materials appendix provides additional technical details and proofs.

2. The financial framework

In this section, we specify the general financial framework in which we are considering dependence risk. We review the characteristics of our financial market with two-asset derivatives. We also detail the portfolio and trading universe of the considered agent.

2.1. Setting and assumptions

Our financial market has two risky assets, whose initial prices are $S_0^1 > 0$ and $S_0^2 > 0$. We consider a final time horizon $T < +\infty$. The final prices of the risky assets are non-negative random variables denoted by S_T^1 and S_T^2. The risk free rate r is considered to be constant and the discount factor is computed as e^{-rT}. For modeling purposes we consider the log-returns of the risky assets defined as $Y_T^1 = \ln \frac{S_T^1}{S_0^1}$ and $Y_T^2 = \ln \frac{S_T^2}{S_0^2}$.

In this market, a two-asset derivative is a financial contract whose payoff depends on the final values taken by the risky assets. Let Z be such a derivative and $z : \mathbb{R}^2 \longrightarrow \mathbb{R}$ its payoff function. Its final payoff is $Z_T = z\left(Y_T^1, Y_T^2\right)$, paid at time T.

This setup with two risky assets leads to an incomplete market model. We denote by $\mathcal{Q}$ the set of feasible risk-neutral measures. $\mathcal{Q}$ contains the martingale measures, associated with the money-market account as numéraire, and compatible with the observed prices of risky assets and derivatives written on them. We assume that our market is free of arbitrage so that $\mathcal{Q}$ is non-empty. Here, free of arbitrage can be interpreted as free of mispricing by the agents involved in the market. We also assume that these agents use a risk-neutral probability $\mathbb{Q} \in \mathcal{Q}$ for valuation purposes. So that Z_0, the current price of Z, is computed as a discounted expectation of its final payoff under $\mathbb{Q}$

$$Z_0 = e^{-rT} \mathbb{E}^{\mathbb{Q}}\left[z\left(Y_T^1, Y_T^2\right)\right]. \quad (1)$$

[1] Quantile based measures of risk, such as the Value-at-Risk (VaR), are the standard measures used to compute capital requirements for market risk in the Basel framework for banking regulation.

[2] Also named bistochastic.

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