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An inventory system with demand dependent on both time and price assuming backlogged shortages

Luis A. San-José [a,*], Joaquín Sicilia [b], David Alcaide-López-de-Pablo [b]

[a] IMUVA, Departamento de Matemática Aplicada, Universidad de Valladolid, Valladolid, Spain
[b] Departamento de Matemáticas, Estadística e Investigación Operativa, Universidad de La Laguna, Tenerife, Spain

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ABSTRACT

In this work we analyze an inventory model for items whose demand is a bivariate function of price and time. It is supposed that the demand rate multiplicatively combines the effects of a time-power function and a price-logit function. The aim is to maximize the profit per time unit, assuming that the inventory cost per time unit is the sum of the holding, shortage, ordering and purchasing costs. An algorithm is developed to find the optimal price, the optimal lot size and the optimal replenishment cycle. Several numerical examples are introduced to illustrate the solution procedure.

1. Introduction

In the twenty-first century, with the globalization of the markets, there has been a considerable increase in trade throughout the world. Firms produce, maintain and distribute goods on all continents. Customers demand products that must be supplied quickly and efficiently. The coordination of the production, maintenance and distribution of the products to meet customer demand and not lose market share with respect to other firms, requires an adequate planning and administration of the inventories. Thus, Inventory Management has become a vital activity for companies to successfully compete in business.

Stock management models help to determine the optimal inventory policies that must be implemented to minimize the inherent costs associated with the maintenance and management of products. Some of the most common assumptions in the study of economic order quantity (EOQ) inventory models are to consider a constant demand rate (independent of time and the unit selling price) and to allow no shortages. However, in many real situations, the demand rate is not constant and may be dependent on time and/or the selling price. Stockouts may also occur and this must be permitted in the inventory model.

When demand is dependent on time, there are different ways by which products are withdrawn from stock during the inventory cycle. These shapes are defined as demand patterns. A demand pattern is known as a power pattern if the demand rate depends potentially on the quotient between time and the length of the inventory cycle. Some Inventory systems with a power demand pattern were developed by Naddor (1966). Later, Goel and Aggarwal (1981) and Datta and Pal (1988) studied inventory models with a power demand pattern for deteriorating items. Lee and Wu (2002), Dye (2004), Singh, Singh, and Dutt (2009), Rajeswari and Vanjikkodi (2011) and Mishra and Singh (2013) developed inventory models for deteriorating items with a power demand pattern while also allowing shortages.

In all the above works, the length of the inventory cycle is always known and fixed. However, Sicilia, Febles-Acosta, and González-De la Rosa (2012) analyzed some inventory systems with power demand in which the length of the inventory cycle was not constant but a decision variable. They determined the optimal inventory policy for the system with backlogged shortages and for the system with lost sales.

Chen and Simchi-Levi (2012) described several price-dependent demand functions which may be used in the study of inventory systems. An interesting review of demand functions in decision modeling is published by Huang, Leng, and Parlar (2013). They presented and commented several price-dependent demand functions that have appeared in the literature.

There are several papers on inventory models where demand is a price-dependent function. Thus, Smith, Martínez-Flores, and Cárdenas-Barrón (2007) analyzed an EOQ inventory system with selling price-dependent demand rate. They developed the optimal policy for three specific demand functions. Kocabıyıkoğlu and Popescu (2011) studied the newsvendor problem with

* Corresponding author.
E-mail addresses: augusto@mat.uva.es (L.A. San-José), jsicilia@ull.es (J. Sicilia), dalcaide@ull.es (D. Alcaide-López-de-Pablo).

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price-sensitive demand. Soni (2013) analyzed an inventory model where the demand rate was additive with respect to the stock level and the unit selling price. Wu, Skouri, Teng, and Ouyang (2014) corrected some deficiencies of Soni's model.

Some researchers have analyzed inventory systems where demand depends on time and price. The demand rate is usually a separable function of time and unit selling price. Thus, Avinadav, Herbon, and Spiegel (2014) studied two inventory models with price and time-dependent demand, but without shortages (one with multiplicative influence of price and time, and the other with additive effect).

In this paper, we assume that shortages are allowed and completely backlogged. This assumption is also considered in other papers on inventory systems. Thus, San-José and García-Laguna (2009) presented a composite lot size model with discounts in all units (constant demand) assuming full backlogging. Birbil, Bülbül, Frenk, and Mulder (2015) studied EOQ models with constant demand and purchase-price and transportation cost functions, considering backlogged shortages. Jakšič and Fransoo (2015) developed a dynamic programming model for a finite horizon stochastic capacitated inventory system where shortages are fully backlogged. Mishra, Gupta, Yadav, and Rawat (2015) presented an EOQ inventory model with full backlogging and deterioration in a fuzzy environment.

Economic order quantity replenishment models focus considerable attention in Inventory Control nowadays. Thus, some recent papers on this topic are the following: Muriana (2016), Bakal, Bayındır, and Emer (2017), Demirag, Kumar, and Rao (2017), Dobson, Pinker, and Yildiz (2017), Herbon and Khmelnitsky (2017), and San-José, Sicilia, González-De-la Rosa, and Febles-Acosta (2017).

To the best of the authors' knowledge, there is no published model developing the optimal policy for an inventory system with full backlogging, where the inventory cycle is a decision variable and demand multiplicatively combines the effects of selling price and a power demand pattern, assuming that the demand rate is the product of a price-logit function and a power-time function.

The remainder of the paper is organized as follows. Section 2 presents the notation and the assumptions related to the inventory system here studied. In the next section, the development of the inventory model and the formulation of the optimization problem is shown. Then, we prove several results that derive to an algorithmic approach to optimally solve the inventory problem. Several numerical examples are discussed to illustrate the procedure for solving the inventory problem. Next, a sensitivity analysis on some input parameters associated with the demand rate of the inventory model is presented. Finally, the conclusions of the work are presented and future research areas are suggested.

2. Assumptions and notation

The notations used in this work are shown in Table 1.

In this work, an economic order quantity model is developed under the following assumptions:

1. The inventory system considers a single product.
2. The planning horizon is infinite and the replenishment is instantaneous.
3. The lead time is zero or negligible.
4. The demand rate $D(s, t)$ is a bivariate function of price and time. We suppose that $D(s,t) = d_1(s)d_2(t)$, where $d_1(s)$ is a known logit-function of price and $d_2(t)$ is a power time-dependent function. That is, the demand rate multiplicatively combines the effects of selling price and a power demand pattern.

Table 1
List of notations.

τ_1	Time period where the net stock is positive (≥ 0)
τ_2	Time period where the net stock is less than or equal to zero (≥ 0)
T	Length of the inventory cycle, that is, $T = \tau_1 + \tau_2$ (> 0, *decision variable*)
M	Maximum level of the stock (≥ 0, *decision variable*)
b	Maximum backlogged quantity per cycle (≥ 0)
Q	Lot size per cycle, that is $Q = M + b (> 0)$
p	Unit purchasing cost (> 0)
s	Unit selling price ($s \geq p$, *decision variable*)
K	Ordering cost per replenishment (> 0)
h	Holding cost per unit and per unit time (> 0)
ω	Shortage cost per backordered unit and per unit time (> 0)
$D(s, t)$	Demand rate at time t when the selling price is s, with $0 < t < T$
$I(s, t)$	Inventory level at time t when the selling price is s, with $0 \leq t < T$
n	Demand pattern index (> 0)
$B(s, M, T)$	Total profit per unit time

5. The order cost is fixed regardless of the lot size.
6. The holding cost per unit is a linear function of time in storage.
7. The system allows shortages, which are completely backlogged.
8. There is single procurement of size Q units to the start of inventory cycle and is equal to the total demand throughout the inventory cycle.

3. Model development

In this work, a continuous review inventory system over an infinite-horizon with deterministic demand is analyzed. It is assumed that shortages are completely backlogged.

At the beginning of the inventory cycle there are M units in the stock. That amount meets demand during the time period $(0, \tau_1]$. Thus, we have

$$M = \int_0^{\tau_1} D(s, u) du.$$

Next, the inventory falls into shortage because there is not enough stock to meet demand. During the time period (τ_1, T), shortages are accumulated and fully backlogged. Thus, from $t = 0$ to T time units, the inventory level decreases due to demand. So, the net stock level $I(s, t)$ is a T-periodic function defined on the interval $[0, \infty)$. The net stock level at time t is given by

$$I(s,t) = M - \int_0^t D(s,u)du = \int_t^{\tau_1} D(s,u)du = d_1(s) \int_t^{\tau_1} d_2(u)du.$$

We suppose that $d_1(s)$ is the logit function given by

$$d_1(s) = \frac{\alpha e^{-\beta s}}{1 + e^{-\beta s}}, \quad \text{with } \alpha > 0 \text{ and } \beta > 0.$$

The parameter α represents the market size and the parameter β is a coefficient of the price sensitivity. The function $d_2(t)$ is a power time-dependent function given by

$$d_2(t) = \tfrac{1}{n}\left(\frac{t}{T}\right)^{(1-n)/n}, \quad \text{with } n > 0.$$

A justification of the practical utility of these functions $d_1(s)$ and $d_2(t)$ to describe the demand for certain products can be seen, respectively, in Sudhir (2001) and San-José et al. (2017).

Therefore, the net stock level at time t is

$$I(s,t) = \frac{\alpha e^{-\beta s}}{1 + e^{-\beta s}} T\left[\left(\frac{\tau_1}{T}\right)^{1/n} - \left(\frac{t}{T}\right)^{1/n}\right] = M - \frac{\alpha e^{-\beta s}}{1 + e^{-\beta s}} T\left(\frac{t}{T}\right)^{1/n}. \quad (1)$$

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