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Improved exact approaches for row layout problems with departments of equal length☆

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ABSTRACT

Facility layout is a well-known operations research problem that arises in various applications. The multi-row layout is a challenging optimization problem where the task is to determine the optimal placement of one-dimensional departments on a given number of rows. This paper is concerned with multi-row facility layout problems in which all the departments have the same length. This is an important special case that includes most multi-row facility layout applications from the literature. We prove two theoretical results about the structure of optimal layouts, namely that only spaces of unit length are necessary to obtain an optimal solution, and that exact expressions exist for the minimum number of such spaces that need to be added so as to preserve at least one global optimal solution. Using these results we propose a binary linear optimization model and a binary semidefinite optimization model for the problem, neither of which uses continuous variables, which has a significant positive computational impact. Our computational experiments show that our specially tailored approaches can handle much larger instances than other exact methods applicable to this important problem class.

1. Introduction

Facility layout is a well-known and challenging operations management problem that arises in various applications. The task is to determine an optimal placement of departments inside a plant according to a given objective function. This function usually reflects the transportation costs (for the material flow) as well as the construction cost of an associated material-handling system. Operations research techniques are often used to solve the facility layout problem effectively.

The exact solution of facility layout problems is generally extremely challenging even for relatively small instances, see e.g., Anjos and Vieira (2017). For this reason it is common to restrict the shape of the layout or of the associated path system. For example, in the single-row facility layout problem (SRFLP), all the departments are placed in a single row, i.e., on one side of a straight path. In this case there always exists an optimal solution such that there are no spaces between neighboring departments. The SRFLP is well-studied. Currently instances of the SRFLP with up to 42 departments can be solved to optimality in reasonable time (Hungerländer & Rendl, 2013). For further details on a variety of both exact methods and heuristics for the SRFLP we refer the reader to the surveys (Anjos & Liers, 2012; Keller & Buscher, 2015; Kothari & Ghosh, 2012).

1.1. Multi-row facility layout and related problems

The multi-row facility layout problem (MRFLP) is an extension of the SRFLP in which the departments can be placed in two or more parallel rows. In contrast to the SRFLP where there always exists an optimal solution without additional spaces between neighboring departments, the optimal layouts for MRFLP may include spaces between neighboring departments in the same row or at the margins of the rows.

Given d one-dimensional departments $\{1, \ldots, d\} = [d]$ with given positive lengths $\ell_1, \ldots, \ell_d$, pairwise non-negative weights w_{ij} indicating the (material) flow between each pair $i, j \in [d]$, $i < j$, of departments, and a set $\mathcal{R} := \{1, \ldots, m\} = [m]$ of rows available for placing the departments, the objective of the MRFLP is to determine

1. an assignment $r : [d] \to \mathcal{R}$ of departments to rows, and

☆ A short paper containing a brief outline of the ideas specialized to the double-row problem, without any technical details, appeared in the proceedings of the OR 2014 (Anjos, Fischer, and Hungerländer, 2016).

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2. a function $p:[d]\to\mathbb{R}$ such that $|p(i)-p(j)|\ge\frac{1}{2}(\ell_i+\ell_j)$ if $r(i)=r(j),\ i\neq j$, i.e., horizontal positions for the centers of the departments within each row without overlap,

so that the total weighted sum of the center-to-center distances between all pairs of departments is minimized. The MRFLP can thus be formulated as the following optimization problem:

$$\min_{r,p} \sum_{\substack{i,j\in[d]\\ i<j}} w_{ij}|p(i)-p(j)|$$

$$\text{s.t.}\quad |p(i)-p(j)|\ge\tfrac{1}{2}(\ell_i+\ell_j),\quad i,j\in[d],\ r(i)=r(j),\ i\neq j.$$

Note that even if we know the assignment r of the departments to the rows and the order of the departments in each of the rows, the exact positions p of the departments are not clear since there might be spaces between neighboring departments in the same row. But given this information, the positions can be determined by solving a linear program.

Inter-row (vertical) distances between the departments are neglected in the above objective function, and we assume that each department can be placed next to another department without any clearance restrictions. If $|\mathcal{R}|=2$, then there are two rows of departments with a straight path between them. We denote this important special case as the double-row facility layout problem (DRFLP). Fig. 1 shows an example of a layout with three rows and seven departments, where $r(1)=r(2)=r(3)=1,\ r(4)=r(5)=2,\ \ r(6)=r(7)=3$ and $p(1)=p(4)=p(6)=1,\ \ p(2)=2,\ p(3)=p(5)=p(7)=3$.

Various applications and extensions of the MRFLP have been studied, see, e.g., Heragu and Kusiak (1991), Gen, Ida, and Cheng (1995), Wang, Zuo, Liu, Zhao, and Li (2015), Zuo, Murray, and Smith (2014), Tang, Zuo, Wang, and Zhao (2015), and Zuo, Murray, and Smith (2016). Somewhat surprisingly, the development of exact approaches to the MRFLP has received limited attention in the literature. Heragu and Kusiak (1988) proposed a nonlinear programming model and obtained locally optimal solutions to the SRFLP and the DRFLP. More recently, Chung and Tanchoco (2010) (see also Zhang & Murray, 2012) focused on the double-row problem and proposed a mixed integer linear programming (MILP) formulation that was tested in conjunction with several heuristics. They solved instances with up to 10 departments within 10 minutes. Amaral (2013a) proposed an improved MILP formulation that solves instances with up to 12 departments. With an extended model (Secchin & Amaral, 2014) were able to solve instances with up to 15 departments in at most eleven hours. Hungerländer and Anjos (2015) put forward a semidefinite programming (SDP) approach for the general MRFLP that can solve instances with fewer than 12 departments to global optimality. Recently, Fischer, Fischer, and Hungerländer (2015) were able to solve DRFLP instances with up to 16 departments to optimality by iteratively using MILPs in an enumerative scheme.

Due to the challenging nature of the MRFLP, several simpler (but still $\mathcal{NP}$-hard) variants of the MRFLP have been considered in the literature. In each of the variations some of the following three decisions are fixed in advance: one has to determine the row of each department, the order of the departments in each row and the exact positions of the departments within the rows since there might be spaces. For example, in the space-free multi-row facility layout problem, spaces between the departments or at the left margin of the rows are forbidden. So knowing the row of each department and the order of the departments in each row suffices for solving this variant. The special case $m=2$ of space-free row layout is also known as the corridor allocation problem, and Hungerländer and Anjos (2016) used an SDP approach that provides high-quality global bounds for space-free double-row instances with up to 15 departments and for space-free multi-row instances with up to 5 rows and 11 departments. Amaral (2012) proposed an MILP formulation for the corridor allocation problem that is able to solve space-free instances with up to 13 departments. Recently, Fischer et al. (2015) and Fischer, Fischer, and Hungerländer (2017) were able to solve space-free double-row instances with up to 16 departments.

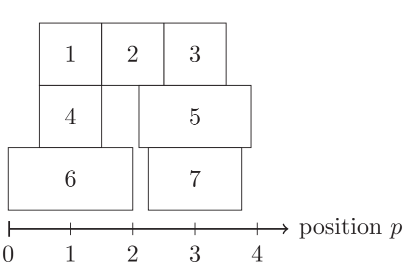

Fig. 1. Illustration of a layout with three rows and seven departments.

The parallel row ordering problem (Amaral, 2013b; Fischer et al., 2015; 2017; Hungerländer & Anjos, 2016) is again a special case of the space-free MRFLP with the additional assumption that the assignment r of departments to rows is already given. So only the order of the departments in each row has to be determined. Exact solutions for instances with up to 25 departments can be determined by an MILP model in Fischer et al. (2015). This MILP is also called iteratively in the enumeration scheme of the currently best DRFLP solver (Fischer et al., 2015). An SDP approach (Hungerländer & Anjos, 2016) allows deriving good lower and upper bounds for instances with two to five rows and up to 100 departments.

1.2. Multi-row facility layout with departments of equal length

This paper is concerned with the special case of the MRFLP with departments of equal length, denoted (MREFLP), in which spaces are allowed, the row assignments are not given, and all department lengths are equal. The MREFLP is also known as the equidistant MRFLP, where we set w.l.o.g. $\ell_i=1,\ i\in[d]$. The MREFLP can also be interpreted as an extension of the classical $\mathcal{NP}$-hard (Garey, Johnson, & Stockmeyer, 1976) (weighted) linear arrangement problem (Díaz, Petit, & Serna, 2002), where at most m nodes are assigned to one position. Hence the MREFLP is also $\mathcal{NP}$-hard.

The case of SRFLP with departments of equal length (SREFLP) has been studied before, and it turns out that the best models for the general SRFLP are also the best ones for the SREFLP (Hungerländer, 2014). This is not the case for the MREFLP, and we show in this paper that it is possible to exploit the additional problem structure for the development of tailored approaches.

Amaral (2011b) proposed an MILP formulation for the minimum duplex arrangement problem, which in our terminology is denoted as DRFLP with departments of equal length (DREFLP). His approach exploits the sparsity of the instances considered and is able to solve randomly generated instances with at most 10 departments (for dense instances) to 20 departments (for extremely sparse instances). Amaral's MILP is closely related to models for the $\mathcal{NP}$-hard Quadratic Assignment Problem (QAP) that is known to be a particularly challenging combinatorial optimization problem in practice. The QAP asks for an assignment of n facilities to n locations that minimizes the sum of the distances between pairs of locations multiplied by the corresponding flows between pairs of facilities. For further details see, e.g., the survey paper (Loiola, de Abreu, Boaventura-Netto, Hahn, & Querido, 2007) and the book (Burkard, Dell'Amico, & Martello, 2009).

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