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Discrete Optimization

A beam search approach to solve the convex irregular bin packing problem with guillotine guts

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ABSTRACT

This paper presents a two dimensional convex irregular bin packing problem with guillotine cuts. The problem combines the challenges of tackling the complexity of packing irregular pieces, guaranteeing guillotine cuts that are not always orthogonal to the edges of the bin, and allocating pieces to bins that are not necessarily of the same size. This problem is known as a two-dimensional multi bin size bin packing problem with convex irregular pieces and guillotine cuts. Since pieces are separated by means of guillotine cuts, our study is restricted to convex pieces.A beam search algorithm is described, which is successfully applied to both the multi and single bin size instances. The algorithm is competitive with the results reported in the literature for the single bin size problem and provides the first results for the multi bin size problem.

1. Introduction

This paper tackles the two-dimensional (2D) multi bin size bin packing problem with irregular pieces and guillotine cuts (MBSBPPGC) and can also solve the single bin size version of the problem (SBSBPPGC). Guillotine cuts arise due to the cutting process of certain materials, where cuts are restricted to extend from one edge of the stock sheet to another. Research on guillotine cuts can be divided into two main groups: one where all the pieces are rectangular, thus the obvious cuts are orthogonal to the stock sheet; and the other where pieces are irregular convex polygons. For this case, allowing both orthogonal and non-orthogonal cuts are the best option for minimising the waste. Our research focuses on the second, less studied, version of the problem. The algorithm we present in this research successfully solves two versions of the problem; the multi bin size and the single bin size problem. This extends the literature in this problem, which only tackles the single bin size version.

The 2D-cutting and packing problem with guillotine cuts and rectangular pieces was first introduced by Gilmore and Gomory (1965) and has been widely studied ever since. Lodi, Martello, and Vigo (1999b) survey 2D rectangle bin packing, including algorithms that handle guillotine constraints. Their approach consists of packing pieces onto shelves along the width of the bin. Lodi, Martello, and Vigo (1999a), Lodi, Monaci, and Pietrobuoni (2017) consider the case where no rotation is allowed. Charalambous and Fleszar (2011) use an alternative approach and generate patterns across the width of the bin, and then within the free rectangle areas. Pieces may be rotated in order to maximise the area of the biggest free rectangle. Fleszar (2013) proposes a constructive heuristic where the insertion decision is made by first-fit, best-fit or critical-fit criteria. Note that in most cases, when dealing with rectangle cutting the number of transitions between horizontal and vertical cuts, called stages, is restricted. At each stage several guillotine cuts can be performed. The problem is known as the n-stage two dimensional bin packing problem, see Puchinger and Raidl (2007), Alvelos, Chan, Vilaca, Silva, and de Carvalho (2009) or Malaguti, Durán, and Toth (2014).

When cutting more complex shapes, cutting problems face new challenges due to the geometry of the pieces. The problem is then known as an irregular packing problems. For many years, the literatures focused on the open dimension version of the problem, called irregular strip packing problems. A useful review of methods can be found in Bennell and Oliveira (2009). Moreover, the solution approaches were largely heuristic. More recently researchers have been developing exact methodologies for these problems. One of the recent exact methods for irregular strip packing is the one from Cherri et al. (2016) where they apply a mixed integer linear programming model. Their model builds on the linear programming compaction approaches (for example Bennell & Downsland, 2001) and adds binary variables to activate and deactivate

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constraints. Larger problems are solved by Rodrigues and Toledo (2017) where their integer programming model approximates the stock sheet by discrete points. A compromise between the two approaches is proposed by Leao, Toledo, Oliveira, and Carravilla (2016) who discretise the stock sheet in the y-axis but allow continuous translations in the x-direction.

A further advancement in the literature is the consideration of multiple fixed dimension stock sheets; irregular bin packing. This is the problem solved by Martinez-Sykora, Alvarez-Valdes, Bennell, Ruiz, and Tamarit (2017) using exact and heuristic methods. This problem is also tackled by Abeysooriya, Bennell, and Martinez-Sykora (2018) who apply a pure heuristic. Both these papers allow any angle of rotation of the pieces. The irregular packing literature provide important findings to consider in our algorithm design, but we point out that none of these papers consider guillotine constraints.

Han, Bennell, Zhao, and Song (2013) introduce the irregular bin packing problem with guillotine cuts into the literature, and propose a one-stage approach that matches pieces into clusters, enclosed in their convex hull to create a block. The clusters are built using a forest search structure where the forest is populated by matching pieces with pieces, pieces with clusters, and clusters with clusters. The forest is complete once there are no further matches for the blocks. The bin packing sequentially selects the block with the greatest summed area of pieces, while removing blocks that contain common pieces to those already selected.

Martínez-Sykora, Álvarez-Valdés, Bennell, and Tamarit (2015) is the only other paper, to the best of our knowledge, that tackles the problem considered in this research. They use a metaheuristic where the relative position of pieces in the bin and guillotine constraints are calculated via mixed integer programming (MIP). The MIP consists of containment constraints, non-overlapping constraints and guillotine cut constraints. It not only determines whether a piece can feasibly be inserted into the bin, but also where to place it. Every time a new piece is inserted, the MIP may change the position of the pieces already in the bin, respecting the guillotine cut structure defined in previous steps. The formulation assumes that pieces are orientated at a certain angle. Hence, the model is solved several times for each piece to be inserted, trying different rotation angles and the reflection of the piece. Pieces are sorted by a certain criterion, and if the next piece in the list does not fit in the current bin, the algorithm tries to insert any of the remaining pieces before opening a new bin. Clearly, since they solve a MIP for each piece inserted in a bin, and the number of constraints grows with the number of pieces in the bin, this heuristic slows down when dealing with instances where there are many pieces per bin.

The authors present new lower bounds, and some of the results obtained are proven to be optimal, improving those on Han et al. (2013). The lower bounds are determined by a simple MIP model that minimises the number of bins needed to allocate all the pieces where only the area of the pieces and the area of the bin are considered as the packing constraint.

The contributions of this paper are as follows. It is the first paper that solves a multi bin size (MBS) bin packing problem with convex irregular pieces and guillotine constraints. It also presents an effective beam search algorithm which obtains fast and competitive solutions without the aid of specialist software, so it can provide small businesses a tool for their cutting operations. Our method produces better results for the single bin size (SBS) problem than those shown on Han et al. (2013), and the results are competitive when compared with those obtained in Martínez-Sykora et al. (2015) while having shorter execution times.

The remainder of the paper is structured as follows: Section 2 describes the problem in detail, and introduces the relevant notation that is used in this paper. Section 3 describes the beam search heuristic. First it gives a general overview of the method and, in Section 3.2, it focuses on how the method is applied to solve this particular problem. Sections 4 and 5 detail the main steps of the beam search heuristics, including how a node is generated, and how a global solution is constructed. Computational results are shown in Section 6. The paper ends with summary and conclusions in Section 7.

2. Problem description

The problem objective is to cut a set of pieces from the minimum number of stock sheets, hence it is an input minimization problem. There are sufficient rectangular stock sheets available to meet the demand, and these may be of different sizes. Let $\mathcal{P}$ be the demand set of pieces, and each piece is considered to be unique. According to the typology proposed by Wäscher, Haußner, and Schumann (2007) this is a multiple bin size (MBS) bin packing problem. Further refinements are that all pieces are convex, usually irregular, can be freely rotated and reflected and only guillotine cuts are allowed.

A guillotine cut is defined as a straight line cut that begins at an edge of the stock sheet and ends at another edge. A cut divides the stock sheet creating two component stock sheet with boundaries that can define the start and end of the next guillotine cut. In our case, the cuts are not constrained to be parallel to the edges of the stock sheet. Usually when considering guillotine constraints, pieces must be cut free from the stock sheet with a maximum number of cuts; in our problem there are no limits to the number of cuts. The order of the cuts is important to track in order to successfully execute the cutting plan.

Let $\mathcal{B}$ be the set of bins, and $B \in \mathcal{B}$ denote a particular bin with width W_B and length L_B. We consider T different bin sizes. Let $\mathcal{P}$ be the set of pieces, where $p \in \mathcal{P}$ denotes a piece, which is characterized by an ordered list of vertices $(v_1, \ldots, v_{n_p})$, and let the edges be expressed by $e_j = (v_j, v_{j+1})$, where $j = 1, \ldots, n_p - 1$ and the last edge is $e_{n_p} = (v_{n_p}, v_1)$. Each piece can be freely rotated and reflected.

The objective is to maximise the total bin utilization (U), which is equivalent to minimising total waste. Utilization can be calculated as:

$$U = \frac{\sum_{p \in \mathcal{P}} A_p}{\sum_{B \in \mathcal{B}} A_B} \tag{1}$$

where A_p denotes the area of piece p and A_B is the area of bin B. In practice residual material of a partially packed bin can be reused in subsequent operations. The residual appears when practitioners perform a horizontal or vertical cut to a bin, to separate the packed and unpacked areas. We assume that only the last bin may have a residual. Let L_R be the length of the packed area, and W_R its corresponding width, then the area of the last bin is either $L_R \cdot W_B$ or $L_B \cdot W_R$, whichever is smallest.

3. Beam search

In this section, we explain in detail the beam search heuristic to solve the 2D MBSBPPGC. This heuristic uses a tree search structure of nodes and branches analogous to branch and bound, but only a subset of nodes is evaluated in the search tree. At any level, only the nodes considered to be promising are kept for further branching and the remaining nodes are pruned permanently. Its structure lends itself to modeling problems where the solution of the problem may be constructed sequentially.

Beam search was first applied for scheduling problems in Sabuncuoglu and Bayiz (1999) and Ghirardi and Potts (2005). More recently, it has been applied to cutting and packing problems