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Decision Support

Matrix representations of the inverse problem in the graph model for conflict resolution

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ABSTRACT

Given the final individual stability for each decision maker or an equilibrium of interest, a matrix-based method for an inverse analysis is developed in order to calculate all of the possible preferences for each decision maker creating the stability results based on the Nash, general metarationality, symmetric metarationality, or sequential stability definition of possible human interactions in a conflict. The matrix representations are furnished for the relative preferences, unilateral movements and improvements, as well as joint movements and joint improvements for a conflict having two or more decision makers. Theoretical conditions are derived for specifying required preference relationships in an inverse graph model. Under each of the four solution concepts, a matrix relationship is established to obtain all the available preferences for each decision maker causing the specific state to be an equilibrium. To explain how it can be employed in practice, this new approach to inverse analysis is applied to the Elsipogtog First Nation fracking dispute which took place in the Canadian Province of New Brunswick.

1. Introduction

Conflicts occur whenever two or more decision makers (DMs) having differences in value systems, objectives or preferences, interact in the real world. In fact, each DM in a dispute strives to change the course of the conflict and reach a state of interest such as a more preferred state than the status quo. In order to better represent and analyze conflict, many available models to conflict resolution have been proposed within a broad field called game theory. The normal and extensive forms of the game, which are generally considered to be part of classical game theory, were developed by von Neumann and Morgenstern (1944). Classical game theory is considered to be quantitative in nature because it uses cardinal preferences often expressed as utility values. However, sometimes it is difficult for a DM to determine how much he prefers one state to another. Thus, Howard (1971) designed a fresh approach called metagame analysis which only assumes the availability of relative preference information in which a given DM either prefers one state over another or they are equally preferred. A methodology called conflict analysis put forward by Fraser and Hipel (1979, 1984) was an enhancement and expansion of metagame analysis. The graph model for conflict resolution (GMCR), which is more comprehensive than existing methodologies, was proposed by Kilgour, Hipel, and Fang (1987) and Fang, Hipel, and Kilgour (1993). The above three methodologies are regarded as qualitative techniques because only relative preference information between any two states is assumed. Because of the foregoing and other reasons, GMCR is widely employed by practitioners and researchers for investigating real world conflict in a highly flexible yet simple way (Madani, 2013).

According to the GMCR procedure, the elements used in this approach can be classified into three main parts which are input, analysis, and output (Fang, Hipel, Kilgour, & Peng, 2003; Kinsara, Petersons, Hipel, & Kilgour, 2015b). The primary items in the input part are the DMs, feasible states in the dispute and DMs' relative preferences over the states. Either an individual or a group, such as a company, can be a DM. A DM can control one or more options, each of which can be selected or not by the DM who controls it. A feasible state is formed as a specific selection of options by the DMs. The analysis part is employed to determine whether a given state is stable for a specified DM or not. The state is said to be stable for a DM if the DM cannot reach a more preferred state in the midst of moves and counter movements by other DMs. An

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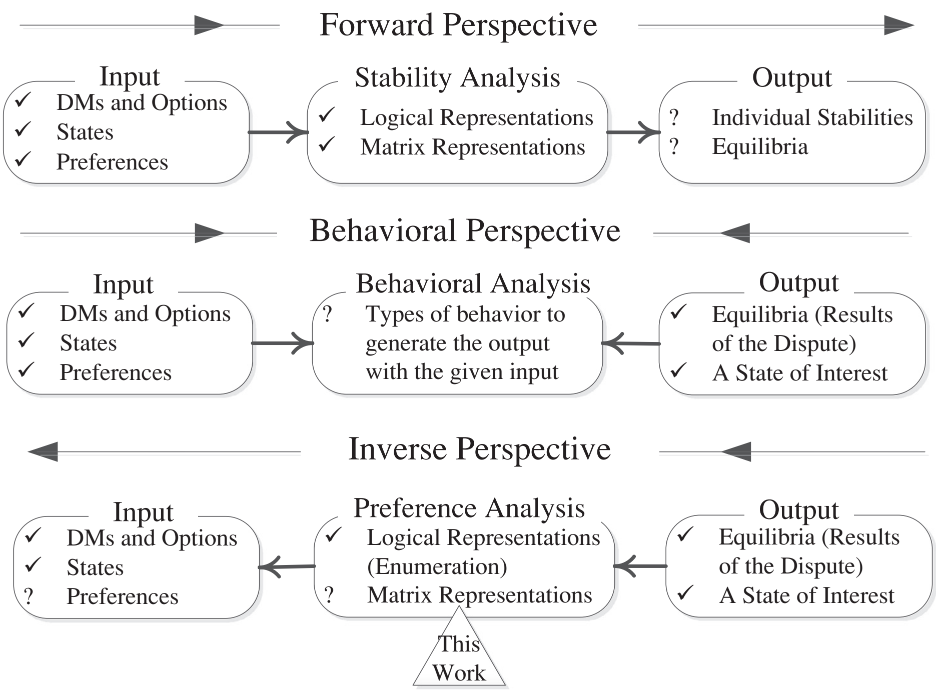

Fig. 1. Three perspectives of carrying out a GMCR study (based on Fig. 1 in the paper by Kinsara et al. (2015b)).

equilibrium of a graph model is a state that is individually stable for all DMs under the same stability definition. A series of stability definitions have been proposed including Nash stability (Nash, 1950, 1951), general metarationality (GMR) (Howard, 1971), symmetric metarationality (SMR) (Howard, 1971), and sequential stability (SEQ) (Fraser & Hipel, 1979, 1984).

The DMs in conflicts may have different purposes when investigating a dispute with different known information. In most situations, one wishes to ascertain the output of an ongoing or a historical dispute by using the analysis engine to calculate various types of individual stability and equilibria after identifying the input part. This is called the forward perspective as portrayed at the top of Fig. 1. In Fig. 1, a check sign (√) means the associated information is known while a question mark (?) indicates an item to determine. Most of the extensions to enrich the theory and applicability of GMCR have been developed under the domain of the forward perspective (Bashar, Kilgour, & Hipel, 2012; Bristow, Fang, & Hipel, 2014; He, Kilgour, & Hipel, 2017; Xu, Hipel, & Kilgour, 2009; Xu, Kilgour, Hipel, & Kemkes, 2010). In some cases, the analyst may wish to determine the type of behavior needed to reach a state of interest. This is called the behavioral problem which is depicted as the middle diagram in Fig. 1 (Kinsara et al., 2015b) and for which a mathematical solution was recently provided (Wang, Hipel, Fang, Xu, & Kilgour, 2018).

In some conflict situations, one wishes to know the preferences required by DMs in order to reach an attractive resolution for all parties. In third party intervention, for example, a third party is invited to a negotiation in order to assist the disputants to reach a win/win resolution (see, for instance, Hipel, Sakamoto, and Hagihara (2015)). The third party facilitators may wish to ascertain which preferences are required by the parties in order to reach such an attractive outcome. In order to analyze the resolution of such conflicts in which the preferences for each DM are unknown or partially unknown, the inverse analysis in a graph model is proposed as displayed at the bottom of Fig. 1. As introduced by Kinsara, Kilgour, and Hipel (2015a), the main feature of the inverse analysis is that the preference information must be determined.

In summary, GMCR can be categorized into three perspectives based on the different given information and goals. As can be appreciated, each perspective solves a different kind of conflict problem. The differences among these three perspectives in a graph model are encapsulated as follows:

(a) The forward perspective determines the possible equilibria by carrying out the stability analysis based on the preferences of each DM contained in the input.
(b) The behavioral perspective ascertains the types of behavior which can produce the outcome of that dispute with the known preferences.
(c) The inverse perspective determines the unknown or partially unknown preference relationships for each DM which are required to make a state of interest be an equilibrium under a specific type of behavior.

Inverse analysis can provide all of the possible preferences for the DMs to reach a desired resolution. For instance, a third party in a conflict (Hipel et al., 2015), who may be a mediator or analyst, can use the results of the inverse approach to determine how to persuade each DM to select the options resulting in the desired equilibrium according to the needed preferences. In other words, a third-party intervenor can employ inverse analysis to design his mediation strategy based on the required preference relationships to reach a more desired outcome. On the other hand, a particular DM involved in the dispute can take advantage of inverse analysis to change his own preferences and attempt to influence a competitor such that an equilibrium of interest can be reached (Kinsara et al., 2015a). In fact, within engineering and science, inverse analysis is referred to as inverse engineering and constitutes a crucial field of study when addressing physical systems problems (Gladwell, 2005). The topic of this paper is inverse engineering within societal systems in the presence of conflict.

In the field of conflict resolution, techniques for tackling the inverse problem possess some drawbacks. More specifically, the inverse model studied by Kinsara et al. (2015a) assumes the employment of ordinal preferences which mean the preferences are