Inconsistency in the ordinal pairwise comparisons method with and without ties

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Inconsistency in the ordinal pairwise comparisons method with and without ties

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Abstract

Comparing alternatives in pairs is a well-known method used to create ranking. Experts are asked to perform a series of binary comparisons and then, using mathematical methods, the final ranking is constructed. Experts conduct a series of single assessments, however, they may not always be consistent. The level of inconsistency among individual assessments is widely accepted as a measure of the ranking quality. The higher the ranking quality, the higher its credibility. One of the earliest and most widespread inconsistency indices is the consistency coefficient defined by Kendall and Babington Smith. In their work, the authors consider binary pairwise comparisons, i.e., those where the result of an individual comparison can only be better or worse.

In the presented work, the maximal number of inconsistent triads in the set of ordinal pairwise comparisons with ties of arbitrary size is determined (formula 14). This, in turn, opens the possibility of effectively extending the Kendall and Babington Smith index to pairwise comparisons, where the result of an individual comparison can be: better, worse or equal. Hence, this effectively extends the use of this index to the Analytic Hierarchy Process and other quantitative methods based on comparing alternatives in pairs. The work also introduces the notions of a generalized tournament and a double tournament as graphs that model ordinal pairwise comparisons with ties and the maximally inconsistent set of pairwise comparisons with ties, respectively. The relationship between the most inconsistent set of pairwise comparisons with ties and the set cover problem is also shown.

Keywords: decision support systems, pairwise comparisons, inconsistency, AHP, set cover problem

1. Introduction

The use of pairwise comparisons (PC) to form judgments has a long history. Probably the first to formally define and use pairwise comparisons for decision making was *Ramon Llull* (the XIII century) [8]. He proposed a voting system based on binary comparisons. The subject of comparisons (alternatives) were people - candidates for election. Voters evaluated the candidates in pairs, deciding which one was better. In the XVIII century, *Llull's* method was

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