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Interfaces with Other Disciplines

Disentangling wrong-way risk: pricing credit valuation adjustment via change of measures

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In many financial contracts (and in particular when trading OTC derivatives), participants are exposed to counterparty risk. The latter is typically rewarded by adjusting the "risk-free price" of derivatives; an adjustment known as *credit value adjustment* (CVA). A key driver of CVA is the dependency between exposure and counterparty risk, known as *wrong-way risk* (WWR). In practice however, correctly addressing WWR is very challenging and calls for heavy numerical techniques. This might explain why WWR is not explicitly handled in the Basel III regulatory framework in spite of its acknowledged importance. In this paper we propose a sound and tractable method to deal efficiently with WWR. Our approach consists in embedding the WWR effect in the drift of the exposure dynamics. Even though this calls for infinite changes of measures, we end up with an appealing compromise between tractability and mathematical rigor, preserving the level of accuracy typically required for CVA figures. The good performances of the method are discussed in a stochastic-intensity default setup based on extensive comparisons of expected positive exposure (EPE) profiles and CVA figures produced (i) by a full bivariate Monte Carlo implementation of the initial model with (ii) our drift-adjustment technique.

1. Introduction

The 2008 financial crisis stressed the importance of taking into account counterparty risk in the valuation of OTC transactions, even when the latter are secured via (clearly imperfect) collateral agreements. Counterparty default risk calls for a price adjustment when valuing OTC derivatives, called the *credit value adjustment* (CVA). Of course, CVA can also apply to other types of bilateral contracts, for example to longevity swaps, see e.g. Biffis, Blake, Pitotti, and Sun (2016). This adjustment depends on the traded portfolio Π and the counterparty C. It represents the market value of the expected losses on the portfolio in case C defaults prior to the portfolio maturity T. Alternatively, this can be seen as today's price of replacing the counterparty in the financial transactions constituting the portfolio, see for example Brigo and Mercurio (2006), Brigo, Morini, and Pallavicini (2013b), Gregory (2010), Stein and Pong (2011), Vrins and Gregory (2011). Interestingly, CVA can also be regarded as a fixed-point problem, as recently shown in Kim and Leung (2016). The mathematical expression of this adjustment can be derived in a rather easy way within a risk-neutral pricing framework. Yet, the computation of the resulting conditional expectation poses some problems when addressing wrong-way risk (WWR), namely accounting for the possible statistical dependence between exposure and counterparty credit risk. Several techniques have been proposed to tackle this point. At this time, there are two main approaches to handle WWR: the dynamic approach (either structural or reduced-form) and the static (resampling) approach. The first one provides an arbitrage-free setup and is popular among academic researchers. Unfortunately, it has the major disadvantage of being computationally intensive and cumbersome, which makes its practical use difficult. On the other hand, the second approach does not have a rigorous justification, but has the nice feature of providing the industry with a tractable alternative to evaluating WWR in a rather simple way. In spite of its significance, WWR is currently not explicitly accounted for in the Basel III regulatory framework; the lack of a reasonable alternative to handle CVA is perhaps one of the reasons.

In this paper, we revisit the CVA problem under WWR and propose an appealing way to handle it in a sound but yet tractable way. We show how the CVA *with* WWR can be written as the CVA *without* WWR provided that the exposure dynamics are modified accordingly. This will be achieved via a *set* of equivalent measures

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called *wrong-way measures* (WWM). The practical benefit of this approach will be to make WWR more tractable in a way that may partly or even totally avoid simulations. Indeed, accurate WWR calculations via simulation can be very time consuming and this may discourage practitioners from computing WWR in practice. This makes semi-analytical approaches like the one introduced hereafter of practical interest.

The paper is organized as follows. Section 2 sets the scene and recalls the fundamental CVA pricing formulae with and without WWR. Next, in Section 3, we briefly review the most popular techniques to address WWR. We then focus on the case where default risk is managed in a stochastic intensity (Cox) framework. Section 4 introduces a set of new numéraires that will generate equivalent martingale measures. Under these WWM measures, the CVA problem with WWR takes a form similar to the CVA problem without WWR, provided that we change the measure under which one computes the expectation of the positive exposure at a future point in time. Section 5 is dedicated to the computation of the exposure dynamics under the WWM. Particular attention is paid to the stochastic drift adjustment under affine intensity models. In order to reduce the complexity of the pricing problem, the (stochastic) drift adjustment is approximated by a deterministic function; the WWR effect is thus fully embedded in the exposure's drift via a deterministic adjustment. Finally, Section 6 proposes an extensive analysis of the performances of the proposed approach in comparison with the standard stochastic intensity method featuring Euler-type discretizations of the bivariate stochastic differential equation (SDE) governing the joint dynamics of default intensity (credit spread risk) and portfolio value (market risk).

2. Counterparty risk adjustment

Define the short (risk-free) rate process $r=(r_t)_{t\geq 0}$ and the corresponding bank account numéraire $B_t:=e^{\int_0^t r_s ds}$ so that the deflator $B:=(B_t)_{t\geq 0}$ has dynamics:

$$dB_t = r_t B_t dt.$$

We work in a complete arbitrage-free market, so that there exists a risk-neutral probability measure $\mathbb{Q}$ associated to this numéraire, under which all B-discounted non-dividend paying tradeable assets are martingales. In this setup, CVA can be computed as the $\mathbb{Q}$- expectation of the non-recovered losses resulting from counterparty's default, discounted according to B. More explicitly, let R be the recovery rate of C and V_t be the closeout price of Π at time t[1]. The general formula for the CVA on portfolio Π traded with counterparty C whose default time is modelled via the random variable $\tau>0$, is given by (see for example Brigo & Masetti, 2005):

$$\begin{aligned}\mathrm{CVA} &= \mathbb{E}^B\left[(1-R)\mathbb{1}_{\{\tau\leq T\}}\frac{V_\tau^+}{B_\tau}\right]\\ &= (1-R)\mathbb{E}^B\left[\mathbb{E}^B\left[H_T\frac{V_\tau^+}{B_\tau}\middle|\sigma(H_u,0\leq u\leq t)\right]\right],\end{aligned}$$

where $\mathbb{E}^B$ denotes the expectation operator under $\mathbb{Q}$, $H:=(H_t)_{t\geq 0}$ is the default indicator process defined as $H_t:=\mathbb{1}_{\{\tau\leq t\}}$, and the second equality results from the assumption that R is a constant and from the tower property. The outer expectation can be written as an integral with respect to the risk-neutral *survival probability*

$$G(t):=\mathbb{Q}[\tau>t]=\mathbb{E}^B\left[\mathbb{1}_{\{\tau>t\}}\right].$$

[1] Here, we assume that this corresponds to the risk-free price of the portfolio which is the most common assumption, named "risk free closeout", even though other choices can be made, such as replacement closeout, see for example Brigo et al. (2013b), Durand and Rutkowski (2013).

The survival probability is a deterministic positive and decreasing function satisfying $G(0)=1$ and typically expressed as $G(t)=e^{-\int_0^t h(s)ds}$ where h is a non-negative function called *hazard rate*. In practice, this curve is bootstrapped from market quotes of securities driven by the creditworthiness of C, i.e. defaultable bonds or credit default swaps (CDS). If τ admits a density, the expression for CVA then becomes

$$\mathrm{CVA} = -(1-R)\int_0^T \mathbb{E}^B\left[\frac{V_t^+}{B_t}\middle|\tau=t\right]dG(t). \tag{1}$$

In the case where the discounted portfolio price process V/B is independent of τ, one can drop the condition in the above expectation to obtain the so-called *standard* (or *independent*) CVA formula:

$$\mathrm{CVA}^\perp := -(1-R)\int_0^T \mathbb{E}^B\left[\frac{V_t^+}{B_t}\right]dG(t), \tag{2}$$

where the superscript $\perp$ in general denotes that the related quantity is computed under the independence assumption. The deterministic function being integrated with respect to the survival probability is called the (*discounted*) *expected positive exposure*, also known under the acronym EPE:

$$\mathrm{EPE}^\perp(t) := \mathbb{E}^B\left[\frac{V_t^+}{B_t}\right].$$

Under this independence assumption, CVA takes the form of the weighted (continuous) sum of European call option prices with strike 0 where the underlying of the option is the residual value of the portfolio Π. The reduced-form approach relies on a change of filtrations whose definitions call for clarifications on the model. We shall consider some explicit examples later in Section 5 but already provide some details. We deal with a market composed of "default-free assets" and "defaultable assets". The term "default" implicitly refers to the credit event of party C. We define three filtrations.

The first filtration $\mathbb{F}=(\mathcal{F}_t)_{0\leq t\leq T}$ is defined as the default-free information. It provides enough (and potentially more) information to determine the prices of the default-free assets. Yet $\mathbb{F}$ only provides *partial* information about the market: we assume $\mathcal{F}_t$ is not rich enough to determine whether C defaulted or not prior to t. We then consider a second filtration $\mathbb{H}=(\mathcal{H}_t)_{0\leq t\leq T}$ generated by the default indicator H. It conveys enough information to determine the potential occurrence of counterparty C's credit event: $\mathcal{H}_t=\sigma(H_u,0\leq u\leq t)$. Notice that $\mathbb{F}$ and $\mathbb{H}$ need not be independent: it is only required that $H_t\in\mathcal{H}_t$ but $H_t\notin\mathcal{F}_t$. Finally, we define a third filtration $\mathbb{G}=(\mathcal{G}_t)_{0\leq t\leq T}$ where $\mathcal{G}_t:=\mathcal{H}_t\vee\mathcal{F}_t$ is representing the total information set available at time t. In our context, this can be viewed as all relevant asset prices and/or risk factors. Therefore, H is adapted to $\mathbb{G}$ but not to $\mathbb{F}$. All stochastic processes considered here are thus defined on a complete filtered probability space $(\Omega,\mathcal{G},\mathbb{G}=(\mathcal{G}_t)_{0\leq t\leq T},\mathbb{Q})$ where $\mathbb{Q}$ is the unique risk-neutral measure and $\mathcal{G}:=\mathcal{G}_T$ with T the investment horizon (which can be considered here as the portfolio maturity). In this setup, we assume that our portfolio Π is only composed of "default-free assets" which in turn means that V and V/B are $\mathbb{F}$-adapted processes. For instance, Π can be composed of a risk-free asset, stocks and options but that are not explicitly sensitive to τ, thereby excluding defaultable bonds and CDS written on C.

A key quantity for tackling default is the Azéma $(\mathbb{Q},\mathbb{F})$-supermartingale (see Dellacherie & Meyer, 1975), defined as the projection of the survival indicator H onto the subfiltration $\mathbb{F}$:

$$S_t:=\mathbb{E}^B\left[\mathbb{1}_{\{\tau>t\}}|\mathcal{F}_t\right]=\mathbb{Q}[\tau>t|\mathcal{F}_t].$$

The financial interpretation of S_t is a survival probability at t given only observation of the default-free filtration $\mathbb{F}$ up to t. Formally, the stochastic process S is linked to the survival probability G by

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