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Decision Support

# Combined optimisation of an open-pit mine outline and the transition depth to underground mining

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ABSTRACT

Miners harvest minerals from ore-bodies in the ground by a variety of specialised mining methods, with most falling into the categories of open-pit and underground. Some ore-bodies are harvested by a combination of open-pit and underground methods. In these cases there is often material that could be mined by either method, and an economic choice has to be made. This is referred to as the transition problem and it has received some attention in the mining literature since the 1980s and more recently has had attention in the mathematics literature. The transition problem is complicated by the need in many cases to leave a crown pillar (un-mined rock above the underground mine) and for this crown pillar to have a prescribed shape.

We have developed a method to optimise the design of an open-pit mine, while solving the transition problem and taking into account the need for a crown pillar with a prescribed shape. We base it on an existing method to optimise the design of an open-pit mine, framed as a maximum graph closure problem. Our method introduces non-trivial strongly connected sub-graphs (NSCSs) of the graph, a complication that previous authors on maximum graph closure problems do not appear to have covered. To obviate the need to check every method for compatibility with NSCSs, we reduce the problem to an equivalent problem without them. This has the added advantage of reducing overall processing time in cases where the number of NSCSs is large.



## 1. Introduction

In this introduction we first present some general mining, planning and optimisation background information, then review the literature on the topic of optimising the transition from open-pit to underground.

Mining is a global industry producing around 17 billion tonnes of mineral fuels, metals and industrial minerals annually (Reichl, Schatz, & Zsak, 2016). The vast majority of minerals are extracted by methods falling into the categories of *open-pit* (an open excavation from the surface) and *underground* (a network of tunnels and/or shafts giving access to the minerals underground). Some ore-bodies are harvested by a combination of open-pit and underground methods, either starting with an open-pit followed by underground mining or less commonly, the reverse. A few example Australian mines are Telfer (a Newcrest owned gold mine that transitioned from open-pit to underground); Sunrise Dam (an AngloGold gold mine that transitioned from open-pit to underground) and Golden Grove (an MMG owned copper and zinc mine that transitioned from underground to open-pit) (Rankin, 2013). Before commencing any such transition a project study is conducted in which strategic mine plans are created. The study may take many months or years. An example is the Grasberg copper mine in Indonesia owned by Freeport-McMoRan. Study work on the transition from open-pit to underground commenced some time before 1995 but was not completed until 2008. Construction commenced in 2011, and first production is forecast for 2018. The expected capital cost is US$6b (Freeport-McMoRan, 2016). With such large sums of money at stake, it is very important to correctly set key design parameters, such as the extents of open-pit and underground mines.

Any kind of mine planning exercise relies on having a model of the ore-body in the ground. In the vast majority of cases, the ore-body is represented in a *regular block model*. This is a model in which each block record in a regular three-dimensional grid carries information about the rock type and mineral grades. In this paper, we restrict our interest to regular block models.

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The application of mathematical optimisation to open-pit mine outlines is widespread, beginning in the 1980s with commercial implementations of a graph algorithm developed by Lerchs and Grossmann (1965). This algorithm, commonly referred to as the *LG Algorithm*, provides an exact method to optimise the outline of an open-pit mine, in order to maximise its un-discounted cash value. The LG Algorithm is in wide use in the mining industry, being available from many of the largest mine planning software vendors (Dassault Systèms; Maptek; Hexagon Mining). The problem can also be framed as a maximum flow problem (Picard, 1976). Maximum flow methods in industrial use include the push-relabel algorithm (Goldberg & Tarjan, 1988) (used by Mincom and Minemax); and Hochbaum's (2008)) pseudoflow (used by Dassault, Deswik and Muir & Associates Computer Consultants). Some of these maximum flow implementations provide significantly better performance than LG implementations. For example, Dray (2014) found that Minemax Planner is significantly faster than Dassault Systèms LG Algorithm implementation, particularly for large block models (e.g. models with more than 5 million blocks). Importantly, Dray also found that the two optimisers yielded the same results.

These pit optimisers decide the outline of the pit in order to maximise un-discounted cash values and they can do so for very large and detailed models (hundreds of thousands or millions of blocks). The computing times vary from seconds to a few hours, depending on the number of blocks and the complexity of the constraints applied to the model. However, miners would rather work with discounted cash-flows and maximise net present value (NPV). Accordingly, pit optimisers are almost always used in a structured process that pursues high NPV solutions for a wide range of planning decisions. The process involves running various optimisers multiple times with different data inputs (for example see Whittle, 2011; Hustrulid, Kuchta, & Martin, 2013). With long planning processes (for example the multi-year Grasberg case), and with efficient pit optimisers available, pit optimisation is generally not on the critical path. However fast optimisation still offers the advantage of allowing a wider range of alternatives to be tested and for larger, more detailed models to be used.

There are many different types of minerals deposits, including those with valuable materials such as gold, copper, nickel and diamonds, and for bulk materials such as coal. In this paper, we are generally interested in mines for valuable materials that are extracted from open pits and/or from underground mines. Most open pit mines for valuable materials are amenable to the open pit optimisation methods discussed in this paper.

In underground mines, we limit our interest to mines that employ *stoping* or *caving* methods. We are not concerned with the details of the underground methods as we really only require a model containing underground mine values for each block in the model, representing the benefit of including the block in the underground mine. That value is derived from an underground mine plan. The reason for this limited concern with the details of the underground plan will become apparent later in the paper. However, for the benefit of readers unfamiliar with mining, we provide some general information: in a stoping mine, tunnels are driven into the ore-body, and the ore is blasted with explosives, and removed by mechanical means from constructed drawpoints. The resulting voids may be left open, or they may be backfilled with waste and cement in order to avoid weakening of the surrounding rock. In a caving mine, miners rely on gravity to collapse the rock down to the drawpoints where it is extracted. Since the ground collapses in (sometimes all the way up to the Earth's surface), there should not be any voids left over in a caving mine. In either stoping or caving mines, drawpoints are accessed by networks of shafts and tunnels, which also provide ventilation, and access for exploration and blasting. The three-dimensional shapes that are targeted for mining are called *stopes* in a stoping mine and *caving blocks* in a caving mine, however to avoid confusion with regular blocks discussed in the paper, we will refer to them as *caving polygons*. A collection of blocks in a regular block model can be used to represent a stope or a caving polygon.

The application of optimisation in design of underground mines is less mature than it is for open-pit mines. However a variety of heuristics and exact methods have been developed to separately optimise the outline of a stope (e.g. Alford & Hall, 2009; Bai, Marcotte, & Simon, 2014), the outline of a caving polygon (e.g. Diering, 2012) and tunnels (e.g. Brazil, Grossman, Rubinstein, & Thomas, 2013; Sandanayake, Topal, & Asad, 2015; Sirinanda, Brazil, Grossman, Rubinstein, & Thomas, 2015).

Various authors have tackled the issue of the combined optimisation of open-pit and underground mines and some have focused in particular on the *transition problem* (optimising the economic decision as to where to stop the open-pit and where to start the underground mine). Whittle (1990) incorporated a method into pit optimisation software that takes into account the value that ore has if mined by an underground method. Consider a case in which some blocks can be mined by either the open-pit or underground methods. For any such block, the value used for pit optimisation should be the difference between its open-pit value and its underground value. The assumption underpinning this is that for a block that can be mined by either method, if it is not mined by the open-pit method, it *will* be mined by the underground method. Camus (1992) independently described an approach that will generate equivalent results. We will henceforth refer to this as the *opportunity cost* approach to solving the transition problem ("opportunity cost approach" for short), following the terminology used in the field of economics (for example see McTaggart, Findlay, & Parkin, 2013).

**Definition 1** (Opportunity cost)**.** Let $v^c \in \mathbb{R}$ and $v^d \in \mathbb{R}_{\geq 0}$ be the net values for mutually exclusive alternatives $c$ and $d$. If no other alternative to $c$ has a higher value than $d$, then the *opportunity cost* for $c$ is $v^d$.

In our case, the mutually exclusive alternatives with respect to any given block are to mine it by open-pit method (alternative $c$ in Definition 1), or by underground method (alternative $d$). If we mine a block by open-pit method, we gain its open-pit value ($v^c$), but we lose the value that would have been gained if it had instead been mined by underground method ($v^d$ – the opportunity cost for $c$). When the opportunity cost approach is applied we subtract the underground value from the open-pit value for each block, before doing pit optimisation. When applying this approach, as opposed to optimising a pit without regard for the opportunity cost, the optimised open-pit is almost always smaller. Also, the value of the open pit mine is lower, but the sum of the values of the open-pit and underground mines is maximised. The reason for this is given later in the paper (Section 2.2). There is more than one way to generate a smaller pit using pit optimisation software; for example, it is common to use a technique called *pit parameterisation* to generate a family of pits by flexing the commodity price (for example see Whittle, 2011). However, the pit created using the opportunity cost approach may not match the shape and tonnage of any of the pits created using parameterisation techniques due to the different ways in which the block values are calculated.

Chen, Gu, and Li (2003) described a method similar to the opportunity cost approach. They did not use exact optimisation, but they did include consideration of a *crown pillar*, which the earlier authors had not. A crown pillar is a body of rock left in place above the shallowest part of an underground mine to ensure stability in the surrounding rock. The need for stability is driven by the land use above the underground mine, which in some cases is an open-pit mine. The crown pillar also acts to reduce or avoid the ingress of water to the underground mine and ensure the stability of the