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Interfaces with Other Disciplines

Investment decisions and sensitivity analysis: NPV-consistency of rates of return

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ABSTRACT

Investment decisions may be evaluated via several different metrics/criteria, which are functions of a vector of *value drivers*. The economic significance and the reliability of a metric depend on its compatibility with the Net Present Value (NPV). Traditionally, a metric is said to be NPV-consistent if it is coherent with NPV in signaling value creation. This paper makes use of Sensitivity Analysis (SA) for measuring coherence between rates of return and NPV. In particular, it introduces a new, stronger definition of NPV-consistency that takes into account the influence of value drivers on the metric output. A metric is strongly NPV-consistent if it signals value creation *and* the ranking of the value drivers in terms of impact on the output is the same as that provided by the NPV. The degree of (in)coherence is calculated with Spearman (1904) correlation coefficient and Iman and Conover (1987) top-down coefficient. We focus on the class of AIRRs (Magni 2010, 2013) and show that the average Return On Investment (ROI) enjoys strong NPV-consistency under several (possibly all) methods of Sensitivity Analysis.

1. Introduction

In capital budgeting many different criteria are used for evaluating a project, measuring economic efficiency, and making decisions. Net Present Value (NPV) is considered the most theoretically reliable tool, since it correctly measures shareholder value creation (Brealey & Myers, 2000; Ross, Westerfield, & Jordan, 2011). However, in practice, many other metrics are used; in particular, relative measures of worth such as internal rate of return (IRR), profitability index (PI), modified internal rate of return (MIRR), Return On Investment (ROI), etc. Recently, a more general notion of rate of return, labeled AIRR (Average Internal Rate of Return) has been developed by Magni (2010, 2013), based on a capital-weighted mean of holding period rates. The AIRR approach consists in associating the capital amounts invested in each period with the corresponding period returns by means of a weighted arithmetic mean. Magni (2010, 2013) showed that any AIRR is NPV-consistent: decisions made by an investor who adopts NPV are the same as those made by an investor who adopts AIRR. Magni (2013) showed that many traditional metrics can be viewed as belonging to the class of AIRRs, including IRR, PI, MIRR. As a special case, this approach makes use of the Return On Investment (ROI) to get an *average ROI*, which is the ratio of the total project return to the total invested capital. Whatever the depreciation pattern, the average ROI exists and is unique, it has the unambiguous nature of investment rate, independent of the value drivers, and decomposes the economic value created into economic efficiency (the difference between average ROI and cost of capital) and investment scale (the sum of the committed amounts).

However, while traditional NPV-consistency is important, under uncertainty, an NPV or a rate of return are not the only factors that drive a decision. The investigation of the risk factors that mainly influence the value of the objective function is no less important.

Sensitivity analysis (SA) investigates the variation of an objective function under changes in the key inputs of a model, so aiming at identifying the most important risk factors affecting the output (and, therefore, the decision) and ranking them. There are many different SA techniques (see Pianosi et al., 2016 and Borgonovo & Plischke, 2016) and, given a technique, different objective functions may or may not lead to different results.

This paper positions itself in the interfaces of operational research (OR) and finance. The strict connections between oper-

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ations management and finance were recognized long since (e.g., Small, 1956, Weingartner, 1963, Adelson, 1965, Hespos & Strassman, 1965, Teichroew, Robichek, & Montalbano, 1965a, Teichroew, Robichek, & Montalbano, 1965b, Rivett, 1974, Ignizio, 1976) and scholarly contributions in the field have grown dramatically in the last decades (e.g., Rosenblatt and Sinuany-Stern, 1989, Grubbström & Ashcroft, 1991, Murthi, Choi, & Desai, 1997, Meier, Christofides, & Salkin, 2001, Gondzio & Kouwenberg, 2001, Baesens, Setiono, Mues, & Vanthienen, 2003, Steuer & Na, 2003, Xu & Birge, 2008, Koç et al., 2009, Fabozzi, Huang, & Zhou, 2010, Thomas, 2010, and Seifert, Seifert, & Protopappa-Siekec, 2013).

The relation between OR and finance is bidirectional. On one side, finance provides a rich toolkit of theories, criteria, and methodologies which enable operational managers to better understand the impact of their decisions so as to maximize the shareholders' wealth: "In order to make decisions managers need criteria of goodness, decision tools, and an understanding of the environment in which they operate ... The main elements of this are that the right criterion of goodness is the maximisation of shareholder wealth and that firms operate in something close to a perfect capital market." (Ashford, Berry, & Dyson, 1988). On the other side, operational research sets the aims and scope of financial modeling for managerial purposes: As opposed to finance theory which uses financial modeling for describing the behavior of the "average" investor and deriving the pricing process of financial assets, operational managers use financial modeling from the point of view of an individual decision maker with specific needs, constraints and preferences (Spronk & Hallerbach, 1997). Further, operations research itself provides techniques and tools that may be applied to several finance problems (Board, Sutcliffe, & Ziemba, 2003).

This paper is in line with the bidirectional relation between operations and finance. Specifically, it recognizes the fundamental roles of economic and financial measures of worth such as the NPV and the ROI for decision-making and, at the same time, applies an OR technique (SA) to such financial measures in order to investigate their compatibility. As such, it falls within that strand of the OR literature which makes use of various economic efficiency measures for managerial purposes, including the NPV (e.g., Yang, Talbot, & Patterson, 1993, Baroum & Patterson, 1996, Herroelen, Van Dommelen, & Demeulemeester, 1997, Cigola & Peccati, 2005, Borgonovo & Peccati, 2006a, Wiesemann, Kuhn, & Rustem, 2010, Leyman & Vanhoucke, 2017), the IRR (Nauss, 1988; Rapp, 1980, Hazen, 2003, Hazen, 2009, Hartman & Schafrick, 2004, Dhavale & Sarkis, 2018), the ROI (e.g., Danaher & Rust, 1996, Myung, Kim, & Tcha, 1997, Brimberg & ReVelle, 2000, Brimberg, Hansen, Laporte, Mladenovic, & Urosevic, 2008, Li, Min, Otake, & Van Voorhis, 2008, Menezes, Kim, & Huang, 2015, Magni, 2016) and the return to outlay (Kumbhakar, 2011). This work is strictly linked with some recent methodological papers within this field which evaluate rationality and robustness of various efficiency measures and/or their sensitivity to changes in the key parameters. Specifically, Magni (2015) showed that the average ROI (labeled average ROA) is reliable for measuring economic efficiency in industrial applications; Mørch, Fagerholta, Pantuso, and Rakkec (2017) used the average ROI as the objective function in a problem of renewal of shippings, and compared the results with those obtained from the traditional NPV maximization. Borgonovo and Peccati (2004, 2006b) studied the impact of the key drivers of an industrial project on NPV, IRR, and value at any time. Borgonovo, Gatti, and Peccati (2010) applied SA in a project financing transaction to assess the degree of coherence between NPV and debt service coverage ratio. Talavera, Nofuentes, and Aguilera (2010) applied SA to the IRR of photovoltaic grid-connected systems. Percoco and Borgonovo (2012) applied SA to IRR and NPV and studied the coherence between the two metrics in terms of importance of key drivers.

We investigate the coherence of average ROI and NPV and give a new, more stringent, definition of NPV-consistency (strong coherence), according to which a metric is strongly NPV-consistent under a given SA technique if it is NPV-consistent in the traditional sense and, in addition, the ranking of the project's value drivers (in terms of influence on the output) is the same. If a metric is not NPV-consistent, the degree of inconsistency may be measured by two alternative indices: Spearman (1904) coefficient or Iman and Conover (1987) top-down coefficient.

We find that the average ROI is strongly NPV-consistent under many techniques, even in a strict sense (the relevances of the parameters are the same). As a result, the average ROI is a reliable measure of worth which can coherently be associated with NPV in investment evaluation, assessment of economic efficiency, and decision-making.

The remaining part of the paper is structured as follows. Section 2 presents the average ROI and the notion of NPV-consistency. Section 3 briefly describes some known SA methods and Section 4 introduces the notion of pairwise coherence according to which any two functions are strongly coherent if the ranking of the model parameters coincides. This section shows that, under many SA techniques, a function f and an affine transformation of it share the same (ranking and) relevances of parameters, so they are strongly coherent in a strict sense. Section 5 shows that the average ROI is strongly NPV-consistent in a strict sense under many SA techniques. Some numerical examples are illustrated in Section 6. Some concluding remarks end the paper. (An Appendix is devoted to some other AIRRs, including non-strongly consistent ones such as IRR, MIRR and EAIRR.)

2. AIRR, average ROI, and NPV consistency

Let P be a project and let $\mathbf{F} = (F_0, F_1, \ldots, F_p) \neq \mathbf{0}$ its estimated stream of free cash flows (FCFs), where $F_0 < 0$ is the investment cost and p is the lifetime of the project. Let τ be the tax rate, R_t be the revenues, O_t be the operating costs, and let Dep_t denote depreciation, $t = 1, 2, \ldots, p$. Then,

$$\begin{aligned} F_t &= \overbrace{(R_t - O_t - \text{Dep}_t)(1-\tau)}^{\text{operating profit}} + \text{Dep}_t \\ &= (R_t - O_t)(1-\tau) + \tau \cdot \text{Dep}_t. \end{aligned} \tag{1}$$

Revenues and costs are often estimated in terms of some key inputs such as prices, quantity produced and sold, unit costs, growth rates, etc. There may be several types of costs, such as energy, material, labor, selling, general, and administrative expenses, etc. For example,

$$F_t = \left(q \cdot p_0 (1+g_p)^t - \sum_{j=1}^{s} O_0^j (1+g_{Oj})^t \right)(1-\tau) + \tau \cdot \text{Dep}_t \tag{2}$$

where p_0 denotes the initial price, q denotes the annual quantity sold, O_0^j denotes the initial amount of the j-th item of cost, g_p and g_{Oj} are the growth rates, and s is the number of cost items involved in the project under consideration. Let k be the (assumed constant) cost of capital (COC). We assume that the COC is exogenously fixed by the decision-maker/analyst. It is well-known that net present value (NPV) measures the economic value created: $\text{NPV} = \sum_{t=0}^{p} F_t (1+k)^{-t}$. Therefore, the NPV decision criterion may be stated as follows:

Definition 1. (NPV criterion) *A project creates value (i.e., it is worth undertaking) if and only if the project NPV, computed at the discount rate k, is positive:* $\text{NPV}(k) > 0$.

Let $\mathbf{C} = (C_0, C_1, \ldots, C_n)$ be any vector representing some notion of capital, such that $C_0 = -F_0$ and $C_n = 0$ and let $I_t = F_t + C_t - C_{t-1}$ be the associated return. An AIRR, denoted as $\bar{\imath}$, is defined