European Journal of Operational Research 000 (2017) 1–11

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

A new binary formulation of the restricted Container Relocation Problem based on a binary encoding of configurations

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ARTICLE INFO

Article history:
Received 12 June 2017
Accepted 27 November 2017
Available online xxx

Keywords:
Combinatorial optimization
OR in maritime industry
Integer programming
Container Relocation Problem
Block Relocation Problem

ABSTRACT

The Container Relocation Problem (CRP), also called Block Relocation Problem (BRP), is concerned with finding a sequence of moves of containers that minimizes the number of relocations needed to retrieve all containers, while respecting a given order of retrieval. The restricted CRP enforces that only containers blocking the target container can be relocated. We improve upon and enhance an existing binary encoding and using it, formulate the restricted CRP as a binary integer programming problem in which we exploit structural properties of the optimal solution. This integer programming formulation reduces significantly the number of variables and constraints compared to existing formulations. Its efficiency is shown through computational results on small and medium sized instances taken from the literature.

1. Introduction

Due to limited space in the storage area in maritime ports, containers are stacked on top of each other. The resulting stacks create rows of containers as shown in Fig. 1. If a container that needs to be retrieved (*target* container) is not located at the top and is covered by other containers, *blocking* containers must be relocated. As a result, during the retrieval process, one or more *relocation* moves are performed by the yard cranes. Such relocations (also called reshuffles) are costly for the port operators and result in delays in the retrieval process. The *Container Relocation Problem (CRP)* (also known as the Block Relocation Problem) addresses this challenge by minimizing the number of relocations. The CRP applies to a broad range of two-dimensional storage systems involving containers, boxes, pallets or steel plates.

Mathematical modeling of the CRP usually represents one row with a two dimensional array of size (T, S), where T is the maximum height and S is the number of stacks. Tiers are numbered from bottom (1) to top (T) and stacks from left (1) to right (S). We refer to this array as a *configuration*. The initial configuration has C containers. In order for the problem to always be feasible, we suppose that the triplet (T, S, C) satisfies $0 \leqslant C \leqslant ST - (T-1)$. Indeed, $T-1$ empty slots would be needed in order to relocate a maximum of $T-1$ blocking containers above the target container (see Wan, Liu, & Tsai (2009)). Each container is identified by a unique integer indicating its relative retrieval order. At any point in the retrieval process, the current target container is the container currently in the configuration with the smallest number. Container c is said to be *blocking* if there exists container d in a lower tier of the same stack such that $d < c$.

During the retrieval process, a container can only be retrieved/relocated if it is at the top of its stack, i.e., no other container is blocking it. Most importantly, *a container can only be relocated if it is blocking the target container* (Assumption A_1 in Caserta, Schwarze, & Voß (2012)). The CRP under these assumptions, referred to as *restricted CRP*, involves finding a sequence of moves to retrieve containers $1, 2, \ldots, C$ (respecting the order) with a minimum number of relocations. Fig. 2 provides a simple example of the CRP.

As the problem with and without Assumption A_1 is NP-hard (Caserta et al. (2012)), many heuristics have been developed to solve this problem (see Forster and Bortfeldt (2012) and Jin, Zhu, and Lim (2015)). This paper focuses on finding optimal solutions for the CRP so we restrict the rest of the review to exact approaches. There are several exact methods worth mentioning such as algorithms based on A^* (Zhu, Qin, Lim, and Zhang (2012), Borjian, Galle, Manshadi, Barnhart, and Jaillet (2015), Tanaka and Takii (2016)), branch-and-bound (B&B) approaches (Ünlüyurt and Aydın (2012), Expósito-Izquierdo, Melián-Batista, and Moreno-Vega (2015)) and the abstraction method (Ku & Arthanari (2016a)).

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https://doi.org/10.1016/j.ejor.2017.11.053
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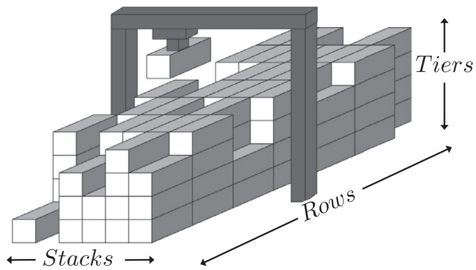

Fig. 1. Stacks of containers in a storage yard.

As the method suggested in this paper is based on solving a binary integer programming (IP) formulation, we review here alternative IP models for the restricted CRP. Wan et al. (2009) formulate one of the first integer programming models for the CRP and develop an IP-based heuristic capable of obtaining near-optimal solutions. Caserta et al. (2012) propose another intuitive formulation of the problem, called BRP-II, as well as an efficient heuristic. Tang, Jiang, Liu, and Dong (2015) propose a very similar formulation with fewer variables than in BRP-II, present heuristics and a worst case analysis. Expósito-Izquierdo et al. (2015) correct BRP-II and rename their new formulation BRP-II*. Eskandari and Azari (2015) also correct BRP-II and propose an improved formulation called BRP2ci by adding valid inequalities. Zehendner, Casserta, Feillet, Schwarze, and Voß (2015) correct and improve BRP-II to get formulation BRP-II-A by removing some variables, tightening some constraints, introducing a new upper bound, and applying a pre-processing step to fix several variables. The two latter formulations being the most recent ones, we compare our new formulation to these state-of-the-art solutions. As we mentioned, these are both improved corrections of BRP-II in Caserta et al. (2012), but they differ in the nature of added cuts as well as the pre-processing step of Zehendner et al. (2015). In addition, the computational results provided by both studies differ. While Zehendner et al. (2015) give results for BRP-II-A on average over set of instances, Eskandari and Azari (2015) only present results on a small subset of instances for BRP2ci, making their comparison difficult. Consequently, we show through our experiments that our new formulation outperforms BRP2ci on available instances from Eskandari and Azari (2015) as well as BRP-II-A on average over sets of instances.

Carlo, Vis, and Roodbergen (2014) and Lehnfeld and Knust (2014) review and survey the existing literature on the CRP and related problems, some of which are mentioned below. The unrestricted version of the CRP relaxes Assumption A_1. Caserta et al. (2012) and Petering and Hussein (2013) develop formulations for the unrestricted CRP, but both are unable to solve small-sized instances efficiently. Tricoire, Scagnetti, and Beham (2018) use an improved B&B to solve the unrestricted problem. Different objective functions have been considered such as the crane travel time, trucks' waiting times or weighted relocations. López-Plata, Expósito-Izquierdo, Lalla-Ruiz, Melián-Batista, and Moreno-Vega (2017) propose a binary IP to minimize waiting times. Priorities could also be given among groups of blocks, and Zhu et al. (2012) and Tanaka and Takii (2016) consider B&B approaches for this case. Ku and Arthanari (2016b) and Galle, Borjian, Manshadi, Barnhart, and Jaillet (2017) study the stochastic version of the CRP, while de Melo da Silva, Erdoğan, Battarra, and Strusevich (2017) introduces another variant of the CRP called the Block Retrieval problem. Borjian, Manshadi, Barnhart, and Jaillet (2013) and Hakan Akyüz and Lee (2014) propose two different MIP formulations for the dynamic version of the CRP where incoming containers have to be stacked. We also mention that Tang, Zhao, and Liu (2012) solve the BRP using integer programming in the case of steel plates, for which our approach can also be applied.

In order to evaluate the efficiency of these methods, several sets of instances have been used. The most common one appears in Caserta, Schwarze, and Voß (2009) and is used by Caserta et al. (2012), Zhu et al. (2012), Petering and Hussein (2013), Borjian et al. (2013), Eskandari and Azari (2015), Expósito-Izquierdo et al. (2015) and Zehendner et al. (2015). In these instances, T and S range from 3 to 10, C is taken to be $(T-2)S$ with $T-2$ containers per stack resulting in 21 classes of problem. With 40 instances per class, this set contains a total of 840 instances. We use these instances in Section 4 to test the efficiency of our new formulation. Instances from Lee and Lee (2010) consider multiple rows and are used to test heuristics. Instances with distinct priorities from Zhu et al. (2012) could be used, but most integer programs are tested on the set of Caserta et al. (2009) hence our preference for this set.

The major contribution of this paper is a new binary IP formulation for the restricted CRP referred to as CRP-I and presented in Section 3. This formulation is novel in several ways. First, CRP-I takes a different modeling approach compared to previous mathematical programming formulations; it builds upon a binary encoding introduced by Caserta et al. (2009) that has never before been used in exact solution methods. We identify and formulate properties of the encoding as linear equalities or inequalities (Section 2.1) in CRP-I and use structural properties of the optimal solution to enhance the tractability of our approach (see Sections 2.2 and 3.1). The simplicity of our formulation and its adaptability to other related problems could be a key enabler of future advances. Finally, we show through extensive computational experiments on small and medium instances that CRP-I improves upon existing mathematical programming formulations by decreasing significantly the number of variables and constraints. In addition, it outperforms most other exact methods on the instances of Caserta et al. (2009) (see Section 4).

This paper is organized as follows. Section 2 introduces some preliminary concepts of the binary encoding and structural properties of the optimal solution. Section 3 presents formulation CRP-I and suggests improvements for this new formulation, such as a revisited pre-processing step. Section 4 presents the results of computational experiments and Section 5 concludes this paper.

2. Preliminaries

2.1. Binary encoding

Most integer program formulations mentioned in Section 1 use the matrix representation of a configuration, i.e., they introduce binary variables of the type y_{tscn} which indicates if container c is in tier t of stack s before the nth move is performed.

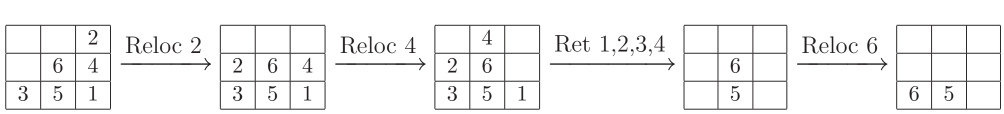

Fig. 2. Configuration for the CRP with 3 tiers, 3 stacks and 6 containers. The optimal solution performs 3 relocations: relocate the container labeled 2 from Stack 3 to Stack 1 on the top of the container labeled 3; relocate 4 from 3 to 2 on the top of 6; retrieve 1; retrieve 2; retrieve 3; retrieve 4; relocate 6 from 2 to the empty Stack 1; retrieve 5; finally, retrieve 6.

Please cite this article as: V. Galle et al., A new binary formulation of the restricted Container Relocation Problem based on a binary encoding of configurations, European Journal of Operational Research (2017), https://doi.org/10.1016/j.ejor.2017.11.053