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Discrete Optimization

A dynamic reformulation heuristic for Generalized Interdiction Problems☆

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ABSTRACT

We consider a subfamily of mixed-integer linear bilevel problems that we call Generalized Interdiction Problems. This class of problems includes, among others, the widely-studied interdiction problems, i.e., zero-sum Stackelberg games where two players (called the leader and the follower) share a set of items, and the leader can interdict the usage of certain items by the follower. Problems of this type can be modeled as Mixed-Integer Nonlinear Programming problems, whose exact solution can be very hard. In this paper we propose a new heuristic scheme based on a single-level and compact mixed-integer linear programming reformulation of the problem obtained by relaxing the integrality of the follower variables. A distinguished feature of our method is that general-purpose mixed-integer cutting planes for the follower problem are exploited, on the fly, to dynamically improve the reformulation. The resulting heuristic algorithm proved very effective on a large number of test instances, often providing an (almost) optimal solution within very short computing times.

1. Introduction

A general bilevel optimization problem is defined as

$$\min_{x\in\mathbb{R}^{n_1}, y\in\mathbb{R}^{n_2}} F(x,y) \tag{1}$$

$$G(x,y) \le 0 \tag{2}$$

$$y \in \arg\max_{y'\in\mathbb{R}^{n_2}} \{ f(x,y') : g(x,y') \le 0 \}, \tag{3}$$

where $F, f : \mathbb{R}^{n_1+n_2} \to \mathbb{R}$, $G : \mathbb{R}^{n_1+n_2} \to \mathbb{R}^{m_1}$, and $g : \mathbb{R}^{n_1+n_2} \to \mathbb{R}^{m_2}$. Let $n = n_1 + n_2$ denote the total number of decision variables.

We will refer to x and y as the *leader* and *follower* decision variable vectors, respectively. Accordingly, $F(x, y)$ and $G(x, y) \le 0$ denote the leader objective function and constraints, while (3) defines the *follower subproblem*. In case the follower subproblem has multiple optimal solutions, we assume that one with minimum leader cost among those with $G(x, y) \le 0$ is chosen, i.e., we consider the *optimistic* version of bilevel optimization (Loridan & Morgan, 1996).

By defining the follower value function for a given leader vector $x \in \mathbb{R}^{n_1}$ as

$$\Phi(x) = \max_{y'\in\mathbb{R}^{n_2}} \{ f(x,y') : g(x,y') \le 0 \}, \tag{4}$$

one can restate the bilevel optimization problem through the so-called *value-function reformulation* (Outrata, 1990) below:

$$\min_{x,y} F(x,y) \tag{5}$$

$$G(x,y) \le 0 \tag{6}$$

$$g(x,y) \le 0 \tag{7}$$

$$(x,y) \in \mathbb{R}^n \tag{8}$$

$$f(x,y) \ge \Phi(x). \tag{9}$$

Problem (5)–(8) is usually called the *High-Point Relaxation* (HPR), which is used in many solution approaches to bilevel programming, including DeNegre (2011), Fischetti, Ljubić, Monaci, and Sinnl (2017).

☆ A dynamic reformulation heuristic for generalized interdiction problems.
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A *Mixed-Integer Bilevel Linear Problem* (MIBLP) is a bilevel problem in which both objective functions $F(x, y)$ and $f(x, y)$ are linear (or affine), and each constraint in $G(x, y) \leq 0$ and in $g(x, y) \leq 0$ is either a linear inequality, or stipulates the integrality of a certain component of (x, y).

Relevant special cases of MIBLPs are known as *interdiction problems* (or interdiction games). They can be seen as a special class of zero-sum Stackelberg games (Von Stackelberg, 1952), i.e., models in which a leader takes some decision and a follower reacts in a sequential way. While in general Stackelberg games the objective functions of the leader and of the follower are different from each other, in interdiction problems the two problems optimize over the same objective function, but in the opposite direction. In addition, in these problems the leader takes decisions that fix some follower variables to zero (Israeli, 1999). Thus, the leader and the follower share a set of items, and the connection between the leader and the follower optimization problems is established through binary "interdiction variables" that are controlled by the leader—a leader solution x being sometimes called *interdiction policy* in this context. Interdiction problems model important applications including marketing (DeNegre, 2011), defense of critical infrastructures (Brown, Carlyle, Salmeron, & Wood, 2005, 2006), and fighting of drug smuggling (Washburn & Wood, 1995) and illegal nuclear projects (Brown, Carlyle, Harney, Skroch, & Wood, 2009; Morton, Pan, & Saeger, 2007).

Contribution. Our main contributions can be summarized as follows.

- We introduce a generalization of the interdiction problems typically addressed in the literature. This variant of problems, to be formally described in Section 3, is obtained by removing the assumption that the leader and follower linear objective functions are one the opposite of the other. Generalized interdiction problems cannot be cast as min–max or max-min problems, so many classical results that hold for standard interdiction problems need to be generalized to the new setting.
- For such generalized interdiction problems, we consider the relaxation obtained by dropping the integrality requirement in the follower, obtaining a bilevel problem that can be reformulated into a standard (i.e., single-level and compact) Mixed-Integer Linear Program (MILP). Similar reformulations have been addressed in the literature for standard interdiction problems both with a MILP follower (e.g., in Caprara, Carvalho, Lodi, & Woeginger, 2016; Lodi, Ralphs, Rossi, & Smriglio, 2011; Lodi, Ralphs, & Woeginger, 2014) and with an LP follower (in this latter case, the reformulation is equivalent to the original problem; see, e.g., Israeli and Wood (2002); Lim and Smith (2007); Wood (1993)). Moreover, for another class of min–max problems, namely the min–max regret robust problems, a related relax-dualise-reformulate idea has been used to devise heuristics (e.g., in Assunção, Noronha, Santos, & Andrade, 2017; Furini, Iori, Martello, & Yagiura, 2015).
  As far as we know, however, the min–max reformulations from the literature were never extended to the generalized interdiction setting we consider in the present paper.
- We describe two basic interdiction heuristics based on the solution of the reformulated problem through a black-box MILP solver, and on the application of refinement procedures to derive a bilevel feasible solution. While these two heuristics are in the spirit of Caprara et al. (2016), Lodi et al. (2011), we propose an improved version based on a dynamic reformulation. This latter algorithm iteratively generates, on the fly, valid cutting planes based on the integrality of the follower variables, thus producing improved reformulations and better solutions. The resulting approach can be viewed as a (row and) column generation method applied to the reformulation, whose effectiveness has been confirmed by extensive computational tests. To the best of our knowledge, a similar mechanism was never used in the context of general-purpose interdiction or min–max heuristics.
- Our heuristic solution scheme can easily be adapted to more general min–max problems, and also to the setting addressed in Israeli and Wood (2002) where interdiction penalties (as opposed to constraints) are considered.
- We report very extensive computational tests on more than 1200 instances both from the literature and randomly generated. This is, by far, the most extensive study on general-purpose interdiction heuristics reported in the literature. The outcome of our experiments is that, although quite simple, even the most basic variant of our heuristic is quite successful for a large number of instances, often providing an optimal solution within negligible computing time. For the hardest instances, however, our more sophisticated version based on the dynamic reformulation gives a significant performance improvement.

The paper is organized as follows. Previous work on interdiction problems and heuristic methods for bilevel (integer) linear programming problems is briefly surveyed in Section 2. The generalized interdiction problem we address in our work is mathematically stated in Section 3, while a single-level compact reformulation of the problem without integrality requirement on the y variables is introduced in Section 4. Section 5 describes our basic heuristic along with two extensions intended to produce improved solutions. Extensive computational results on various classes of test instances are reported in Section 6, while Section 7 draws some conclusions and addresses future directions of work.

2. Previous work

We next focus on previous work regarding interdiction problems. For the sake of completeness, we also briefly outline heuristic approaches for bilevel (integer) linear programming, and refer the reader to, e.g., Fischetti et al. (2017), for previous work on exact approaches for MIBLP.

Exact solvers for interdiction problems. When the follower problem in an interdiction problem can be formulated as linear program, duality-based reformulations give exact algorithms. This idea has been used, e.g., in Wood (1993) for the maximum flow problem, in Lim and Smith (2007) for multi-commodity flow problems, and in Bazgan, Toubaline, and Vanderpooten (2012) for the spanning tree problems. The shortest-path variant for a problem where interdiction does not forbid the use of an item but causes a penalty in the follower objective is studied in Israeli and Wood (2002), where a duality-based exact algorithm is given. Finally, for another variant where interdiction is not a discrete decision, but continuously increases a follower penalty in the objective, duality has also been used in the design of algorithms, see, e.g., Fulkerson and Harding (1977).

A basic interdiction problem with discrete follower is the Knapsack Interdiction Problem (KIP) studied in Caprara et al. (2016), DeNegre (2011), Tang, Richard, and Smith (2016). The problem consists of a Stackelberg game where both the leader and the follower players fill their private knapsacks by choosing items from a common set N. In the first stage, the leader chooses her items subject to her own knapsack capacity (called interdiction budget). In the second step, the follower solves a 0/1 knapsack problem and selects some of the items that are not taken by the leader, with the aim of maximizing the profit of the collected items. The goal of the leader is to obtain the worst possible outcome for the follower. A typical application of this problem arises in marketing, when