European Journal of Operational Research 267 (2018) 187–199

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

Branch-price-and-cut for the Mixed Capacitated General Routing Problem with Time Windows

Claudio Ciancio[a], Demetrio Laganá[a,*], Francesca Vocaturo[b]

[a] Department of Mechanical, Energy and Management Engineering, University of Calabria, Arcavacata di Rende 87036, CS, Italy
[b] Department of Economics, Statistics and Finance, University of Calabria, Arcavacata di Rende 87036, CS, Italy

ARTICLE INFO

Article history:
Received 20 July 2016
Accepted 17 November 2017
Available online 24 November 2017

Keywords:
General routing problem
Graph transformation
Integer programming
Column generation

ABSTRACT

The Mixed Capacitated General Routing Problem with Time Windows (MCGRPTW) is defined over a mixed graph, for which some nodes, arcs and edges have strictly positive demand and must be serviced. The problem consists of determining a set of least-cost vehicle routes that satisfy this requirement, while respecting pre-specified time windows and without exceeding the vehicle capacity. In this work, we transform the MCGRPTW into an equivalent node routing problem over a directed graph. Thus, we solve the equivalent problem by using a branch-price-and-cut algorithm which relies on some effective techniques introduced in the field. Computational experiments over instances derived from the Capacitated Arc Routing Problem with Time Windows and from the Mixed Capacitated General Routing Problem are presented. The article also describes experiments over benchmark instances.

Introduction

The routing problems can be classified in three main classes: node routing problems (also identified as vehicle routing problems), arc routing problems, and general routing problems. In a node routing problem, the customers, i.e. the elements requiring service, correspond to nodes of the graph representing the street network; in an arc routing problem, the customers correspond to links (arcs or edges) of the graph; in a general routing problem, the customers correspond to both nodes and links.

This article deals with a general routing problem, denoted by Mixed Capacitated General Routing Problem with Time Windows (MCGRPTW). To the best of our knowledge, the MCGRPTW has not yet been tackled. It consists of determining a set of least-cost vehicle routes that respect some requirements: (i) each route starts and ends at the depot, where a fleet of identical vehicles is based; (ii) nodes and links with strictly positive demand must be serviced exactly once, while respecting pre-specified time windows; (iii) the total demand collected by a vehicle cannot exceed its capacity. The MCGRPTW arises in classic arc routing contexts. More effectively than the Capacitated Arc Routing Problem (CARP) and the Capacitated Arc Routing Problem with Time Windows (CARPTW), it can model some real world applications in urban waste collection, postal distribution, school bus routing, etc. For instance, within urban waste collection, household garbage quantities are collected along the streets, but specific locations (e.g., industrial and commercial sites, hospitals, schools, and multistorey apartment blocks) need to be considered as single points due to the large amount of garbage in these locations. Adding the complexity of time window constraints into the problem is broadly justified in real-world cases. For example, commercial customers may have time frames; in addition, time windows may be used along the streets to deal with municipal bans on service during peak hours. In this context, the use of a mixed graph is useful to better represent the street network. Specifically, a one-way street corresponds to one arc, a two-way street in which the waste on the two sides must be collected separately corresponds to two opposite arcs, and a two-way street in which the traffic is low and the waste on the two sides can be collected in parallel and in any direction corresponds to one edge.

The literature on node and arc routing problems is much wider than the one on general routing problems, in spite of its interesting real-world applications. However, these latter problems have stimulated more and more contributions from the scientific community in the last years. The aim of this article is to enrich the literature on the general routing problems by increasing their applicability through the introduction of very common operational constraints.

Technically, we transform the MCGRPTW into an equivalent node routing problem over a directed graph. The idea of using graph transformation to solve a routing problem is not new. It was used, for example, by Blais and Laporte (2003); Laporte (1997);

* Corresponding author.
E-mail addresses: claudio.ciancio@unical.it (C. Ciancio), demetrio.lagana@unical.it, demetriosalvatore.lagana@unical.it (D. Laganá), vocaturo@unical.it (F. Vocaturo).

https://doi.org/10.1016/j.ejor.2017.11.039
0377-2217/

Tagmouti, Gendreau, and Potvin (2007). We emphasize that the presence of time windows (especially tight and similar time windows) could increase the number of vehicles required for the service considerably. In the scientific literature, the algorithms based on branch-and-price (B&P) frameworks seem to be less sensitive to the number of vehicles with respect to other algorithms; their performance seems to remain high even if many vehicles are used. In a B&P outline (see Barnhart, Johnson, Nemhauser, Savelsbergh, and Vance, 1998, for an introduction) the classic branch-and-bound method is strengthened by column generation. In short, column generation consists of defining a *master problem* and working only with a sufficiently meaningful subset of variables (columns); then a subproblem, called *pricing problem*, is used to generate improving columns. Optionally, cutting planes can be added in order to strengthen the relaxation at the nodes of the search tree. In this case, the solution framework is also known as branch-price-and-cut (BP&C). There are successful applications of BP&C approaches to generic optimization problems. Besides routing problems (e.g., Desaulniers, 2010; Benavent et al., 2014), we can mention clique partitioning problems (e.g., Ji & Mitchell, 2007), health care planning problems (e.g., Trautsamwieser & Hirsch, 2014), cut packing problems (e.g., Bergner, Lübbecke, & Witt, 2014), project scheduling problems (e.g., Coughlan, Lübbecke, & Schulz, 2015), and many others. This article also presents a BP&C algorithm and makes several contributions to the literature. Specifically: (i) we introduce a new problem, i.e. the MCGRPTW; (ii) we transform it into an equivalent node routing problem before applying the solution approach; the transformation process stands out for several aspects of the MCGRPTW; (iii) we propose test instances for the MCGRPTW; they are expected to be new benchmarks for other researchers.

The remainder of the article is structured as follows. Relevant works in the scientific literature are discussed in Section 1. Section 3.2 provides a formal description of the MCGRPTW and illustrates the transformation of the problem into an equivalent node routing problem. Section 3 introduces suitable elements for column generation. The key components (pricing, branching, cutting) of the BP&C algorithm are described in Section 4. Computational results in Section 5 show the effectiveness of the approach. Finally, Section 6 summarizes the results of the study.

1. Literature review

Among the various topics lied with the MCGRPTW, we mention routing problems under vehicle capacity constraints and routing problems under time windows constraints. In this section, we resume the basic concepts and discuss some pertinent works in both topics.

1.1. Routing problems under vehicle capacity constraints

The solutions of generic routing problems are represented by routes that satisfy operational constraints. One of the most common constraint addressed in the scientific literature concerns the vehicle capacity. In the real applications, the capacity is limited and generally small with respect to the total load which has to be transported; consequently, several routes must be defined in such way that the load over each vehicle does not exceed its capacity. The presence of the capacity constraints increases the computational complexity of the routing problem; in effect, a packing structure must be jointly considered.

The classic version of the problem is known as *Capacitated Vehicle Routing Problem* (CVRP) where a fleet of identical vehicles, all based at a single depot, is available. Moreover, the service activity occurs at the nodes of the graph. The deterministic demands at the nodes are known in advance and cannot be split among different routes. A very recent state-of-the-art of effective exact algorithms for the CVRP is provided by Pecin, Pessoa, Poggi, and Uchoa (2017). These algorithms are based on the combination of column and cut generation. A such combination is also used by Pecin et al. (2017) in an improved solution framework for the problem. There exist many variants of the CVRP. A variant which has received a great attention in the literature arises when the fleet of vehicles is heterogeneous (different capacities and/or costs). The article of Baldacci, Toth, and Vigo (2010) also deals with this case. Another common variant is represented by the multi-depot CVRP, where the vehicles are located at different depots. Recently, Contardo and Martinelli (2014) have presented an exact approach to deal with the multi-depot case, also considering route length constraints. Several (meta)heuristics approaches to the CVRP and its variants are present in the scientific literature (see, for instance, the surveys of Gendreau, Laporte, & Potvin, 2002; Laporte & Semet, 2002).

The CARP is the arc counterpart to the CVRP. CARP variants and methods for their solution are described in the survey of Wøhlk (2008). Interesting contributions on the topic are described in the recent works of Bode and Irnich (2012) and Bartolini, Cordeau, and Laporte (2013). In particular, Bode and Irnich (2012) have proposed a tailored B&P algorithm for the CARP. The first step of this two-phase algorithm is represented by the solution of a specific one-index formulation in order to produce strong cuts and an excellent lower bound. In the second phase, the master problem is initialized with the strong cuts. Bartolini et al. (2013), on their part, have presented a lower bounding method for the problem based on a set partitioning-like formulation with additional cuts. This method uses cut and column generation to solve different relaxations and an effective dynamic programming procedure for generating routes. In addition, the authors have proposed an exact algorithm based on the new lower bounds. In the field of general routing, the *Capacitated General Routing Problem* (CGRP) identifies the topic. For the most popular version of the problem which refers to mixed graphs (MCGRP, *Mixed Capacitated General Routing Problem*), there exist lower bounding procedures (e.g., Bach, Hasle, & Wøhlk, 2013) and non-exact methods (e.g., Prins & Bouchenoua, 2005; Bosco, Laganà, Musmanno, & Vocaturo, 2014; Dell'Amico, Díaz Díaz, Hasle, & Iori, 2016). Moreover, exact algorithms have been presented by Bosco, Laganà, Musmanno, and Vocaturo (2013) and Irnich, Laganà, Schlebusch, and Vocaturo (2015). More recently, Bach, Lysgaard, and Wøhlk (2016) have proposed a different approach, which also uses column generation, to solve MCGRP instances. A preliminary version of their method is described in the Ph.D. thesis of Bach (2014). Variants of the basic problem include the MCGRP with turn penalties (Bräysy, Martínez, Nagata, & Soler, 2011) and the undirected CGRP with profits (Archetti, Bertazzi, Laganà, & Vocaturo, 2017).

1.2. Routing problems under time windows constraints

Many works addressing time windows constraints in node routing problems have been developed in recent years. In practice, the service has to begin within a given time frame, named "time window". It is defined by the earliest time and the latest time. If the vehicle arrives too early, then it must wait. On the contrary, due dates cannot be violated. A brief overview of exact and (meta)heuristic methods for the most common version of the problem, named *Vehicle Routing Problem with Time Windows* (VRPTW), is given by El-Sherbeny (2010). Kallehauge (2008) provides a review of the exact methods for the problem. Analogously, for more details about the (meta)heuristics methods for the VRPTW, we refer the reader to the surveys of Bräysy and Gendreau (2005a); 2005b).

Among the variants of the VRPTW, we mention the problems with multiple time windows and soft time windows. For the first variant, an element requiring service has multiple and disjoint