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Valuation of power plants☆

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ABSTRACT

In this paper we develop continuous-time stochastic control models for valuation and operation of three stylised types of power plants in an electricity market: a renewable plant, a conventional plant and a storage plant. Examples of these types of power plants are respectively wind turbines, gas-fired generation units and hydroelectric facilities. We demonstrate how to derive analytical or quasi-analytical solutions in spite of many details in modeling. In particular, we model uncertainty in electricity prices and in production input/output when it is relevant for the technology considered. Input/output is assumed to follow a diffusion process, whereas the price process may include jumps. Our models account for special characteristics of the technologies, including a non-Normal distribution of wind speeds as well as start-up and shut-down costs of thermal units. We use our models to assess the impact of conjectured future market conditions such as increasing average price level, price volatility and correlation between renewable production and electricity prices.

1. Introduction

With ambitious targets, many future electricity markets will be characterized by large shares of renewable generation such as wind and solar power production. The inability to accurately predict renewable production increases supply uncertainty and thereby the need for flexibility in conventional generation. Evidently, to ensure continued operation of and new investment in generation capacity, production must be profitable. With the current market setup, however, a change in the generation mix will change the dynamics of electricity prices. For instance, the average level and volatility of prices may increase and the correlation between renewable production and prices may become increasingly negative. The future value of power operation and investment may therefore change significantly, depending on generator characteristics.

In this paper we quantify the effects of changes in the price dynamics for three stylized types of power generation: inflexible renewable generation, flexible conventional generation and a storage power plant. Renewable generation, exemplified by a wind turbine, is uncontrollable and cannot adjust to variations in electricity prices. A negative correlation between power production and electricity prices will therefore lower the value of generation and reduce investment incentives. Flexible generation, represented by a gas-fired power generating unit, is controllable. This technology can benefit from periods with high electricity prices, while its operation can be temporarily suspended to avoid periods with low prices. Finally, the storage power plant, exemplified by a hydroelectric facility, can continuously adjust its generation to electricity prices by storing water for periods with high prices to the extent the reservoir capacity allows. For a general introduction to power generation, operation and control, see Wood and Wollenberg (2013).

We assume that renewable generation involves no operational decisions and that production can be modeled by a load factor correlated with the price. For a similar model, see Abadie and Chamorro (2014), in which the price process accounts for seasonality and the valuation problem is solved numerically using a Monte Carlo approach. In contrast, Boomsma, Meade, and Fleten (2012) ignore seasonality and assume constant correlation-adjusted load to solve the problem analytically. In this paper, we likewise derive an analytical expression for the instantaneous value of generation. We model the uncertainty in the load factor with emphasis on capturing its distributional properties in continuous time, including exponentially decaying autocorrelations. To quantify the impact of correlation, however, we directly model wind speeds using the approach of Zárate-Miñano, Anghel, and Milano (2013) and obtain their non-Normal distribution by a transformation of a mean reverting process.

In the absence of start-up costs and operational constraints the value of a gas-fired power plant can be modeled as a sum of spark

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spread call option prices. An example is provided by Deng (1999), who models electricity and fuel prices using mean reverting jump-diffusion processes with either regime switching, deterministic volatility or stochastic volatility and develops quasi-analytic expressions for the option prices. Näsäkkälä and Fleten (2005) model the spark spread directly with a two-factor model and likewise determine quasi-analytic expressions for the value of the gas-fired power plant. When including temporal constraints such as minimum up-time/down-time restrictions or start-up/shut-down costs, the dimension of the problem increases. Tseng and Barz (2002) and Carmona and Ludkovski (2009) solve the operational problem for short time horizons using different combinations of Monte Carlo simulation and dynamic programming, whereas Deng and Oren (2005) and Gardner and Zhuang (2000) apply stochastic dynamic programming to solve the problem on a lattice. An alternative approach is to derive the associated Hamilton-Jacobi-Bellman (HJB) equation and solve this numerically using some finite difference scheme, (see Thompson, Davison, and Rasmussen, 2004 and Safarov and Atkinson, 2017). However, as these methods are subject to the curse of dimensionality, the computation time increases exponentially with increasing accuracy in time and space, making them impractical for longer time horizons. Here, we derive the HJB equation but suggest a one-factor model for the electricity price with mean reversion and jumps over an infinite horizon. This enables us to maximize the option constant and avoid discretization of both time and space.

Valuation of a run-of-river hydroelectric facility resembles that of renewable generation and involves no operational decisions, (see Böckman, Fleten, Juliussen, Langhammer, and Revdal, 2008). As for the conventional power plant, however, operation must be optimised for a reservoir hydropower plant. The storage level introduces another dimension in the valuation problem, which makes analytical solutions significantly more difficult. More specifically, the discharge strategy creates changes in the storage level, or equivalently, the control affects the derivative of an underlying process. This additional complexity likewise arises in the modeling of thermal power plants when including boiler temperature. For short time horizons it is possible to solve the HJB equation numerically, see Thompson et al. (2004) and Chen and Forsyth (2008), who allow for operational constraints such as ramping as well as seasonality in the price process. The combination of Monte Carlo simulation and dynamic programming by Carmona and Ludkovski (2009) is also applied to the hydroelectric power plant, but requires significant computation time. To reduce the running time Näsäkkälä & Keppo (2008) approximates the optimal control strategy using a parametrized boundary and a Monte Carlo approach. Other solution methods rely on linearizations of the operational strategies. For example, Braaten, Gjønnes, Hjertvik, and Fleten (2016) assume the strategies are linear functions of prices, whereas Doege, Schiltknecht, and Lüthi (2006) consider linear combinations of predefined step-functions. These approaches have long time-steps or cover short time horizons. In contrast, we handle operational boundaries of the hydroelectric power plant using penalty functions and linearize the optimal control from the HJB equation to obtain quasi-analytic expressions for the value on an infinite horizon. With this approach we obtain an explicit discharge strategy that is linear in price and storage level and satisfies the storage and flow rate constraints with high probability.

We solve the valuation problems with three price models based on stochastic differential equations. The initial model is a simple shifted Geometric Brownian Motion, which is compared to a shifted exponential Ornstein–Uhlenbeck process that accounts for mean-reversion. To better capture the distributional properties of transformed price increments, the final model is extended to include jumps. These three models are analytically tractable. A comprehensive introduction to electricity price modeling can be found in Benth, Benth, and Koekebakker (2008). For mean reversion and jumps in asset pricing and real options valuation, in general and in energy, see also Hahn and Dyer (2008), Wong and Lo (2009), Tsekrekos and Yannacopoulos (2016) and Castellano, Cerqueti, and Spinesi (2016). For non-Normal electricity spot price modeling and derivatives pricing, see Benth, Kallsen, and Brandis (2007).

The main contribution of the paper can be summarized as follows. We derive solutions for the value of three stylized types of power plants in an electricity market, taking into account their operational characteristics:

- Renewable production cannot be controlled but varies stochastically. Unlike the existing literature, we allow for the non-Normal distribution of such production and the correlation between production and prices, while being able to obtain closed-form solutions to the valuation problem.
- In valuing conventional production, we account for start-up and shut-down costs in operation. To the best of our knowledge, the derivation of quasi-analytical solutions is new to the literature.
- We illustrate how to obtain an analytical solution to the stochastic control problem for hydro power by penalizing deviations from storage and flow rate constraints and assuming a linear control strategy.

The rest of the paper is structured as follows. In Section 2 we develop two diffusion models and a jump-diffusion model for the electricity price and in Section 3 we model input/output uncertainty. Section 4 accounts for correlation between input/output and the electricity price. Section 4.1 considers the value of a wind turbine, Section 4.2 values a gas-fired generating unit and Section 4.3 obtains the value of a hydroelectric facility. In Section 5 we report results for a case study and in Section 6 we study the impact of changes in the price dynamics. Finally, Section 7 provides a brief conclusion.

2. Electricity price uncertainty

We aim to investigate the effects of different electricity price dynamics on the value of power generation and from an investment point of view. We assume that generation is always dispatched in a market and therefore focus on market prices. The market for immediate dispatch of generation is referred to as a spot market and prices are likewise referred to as spot prices, although these are in fact forward prices, typically with a maturity of a day or an hour.

Classical papers on commodity prices such as Schwartz and Smith (2000), Lucia and Schwartz (2002) and Gibson and Schwartz (1990) model the logarithm of the price, $X_t = \log(P_t)$, such that the price has the form $P_t = e^{X_t}$. This captures the skewness of prices when X_t has a symmetric distribution, is analytically tractable and implies that prices are non-negative. Electricity prices may, however, occasionally become negative as a result of unpredictable excess of renewable production combined with insufficient flexibility to quickly reduce conventional generation. Furthermore, this is expected to happen more frequently with increasing shares of renewable production in the electricity market, (see Götz, Henkel, Lenck, and Lenz, 2014). To allow for negative prices we assume that $P_t = e^{X_t} - M$, where $-M$ is a lower bound on prices. With this assumption the model remains analytically tractable. We assume that the logarithm of the shifted price, $X_t = \log(P_t + M)$, follows a stochastic differential equation (SDE) where the dynamics of the price process $(X_t)_{t\geq 0}$ are given by

$$dX_t = \mu(X_t)\ dt + \sigma(X_t)\ dZ_t^P + \gamma(X_t)\ dJ_t.$$

Here, $\mu(X_t)$, $\sigma(X_t)$ and $\gamma(X_t)$ are the drift, diffusion and jump coefficients, respectively. Furthermore, $(Z_t^P)_{t\geq 0}$ is a standard Brownian Motion and $(J_t)_{t\geq 0}$ is a compound Poisson process with