European Journal of Operational Research 000 (2017) 1–14

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Robust reinsurance contracts with uncertainty about jump risk

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ARTICLE INFO

Article history:
Received 2 December 2016
Accepted 28 October 2017
Available online xxx

Keywords:
Game theory
Ambiguity
Proportional reinsurance
Excess-loss reinsurance
Reinsurance price

ABSTRACT

We investigate robust reinsurance contracts in two reinsurance modes, namely proportional reinsurance and excess-loss reinsurance, in a continuous-time principal–agent framework. Insurance claims follow the classic Cramer–Lundberg process. The reinsurer (principal) is concerned about potential ambiguity in the claim intensity, but the insurer (agent) is not. The reinsurer designs a robust reinsurance contract that maximizes the penalty-based multiple-priors utility of terminal wealth, subject to the insurer's incentive compatibility constraint. We derive the analytical expressions of the robust reinsurance contacts. Our results show that the reinsurer dynamically decreases the reinsurance price, which makes the demand for reinsurance increase over time. However, the reinsurer's ambiguity aversion increases the price of reinsurance, which decreases demand. Moreover, the price of excess-loss reinsurance is greater than that of proportional reinsurance. Finally, when the insurer's risk aversion is low or the reinsurer's risk aversion is high, both the insurer and the reinsurer prefer the proportional reinsurance contract.

1. Introduction

Reinsurance offers an insurance company protection by transferring its risk exposure to the reinsurance company. Research on optimal reinsurance dates back to the seminal works of Borch (1960, 1962) and Arrow (1963). These notable works have since been extended and expanded by Blazenko (1986), Bäuerle (2005), Balbás, Balbás, and Heras (2009, 2011), Bi, Meng, and Zhang (2014) and Deng, Zeng, and Zhu (2018), among others. However, most of these studies focus on the insurer's point of view. They assume that the reinsurance price is given, and only consider the insurer's optimal risk allocation problem by optimizing his own objective.

In fact, reinsurance refers to the interests of both the insurer and the reinsurer, and a reinsurance contract includes two basic elements: demand and price. Moreover, the insurer and the reinsurer only behave competitively and take the price as given in a competitive market equilibrium (Kihlstrom & Roth, 1982). However, the reinsurance market is incomplete, which might explain why it is difficult to conduct effective quantitative studies on reinsurance pricing (Gilliam, 1980; Jean-Baptiste & Santomero, 2000). Borch (1969) indicates that a reinsurance contract may be very attractive to one party, but may be quite unacceptable to the other. Then, the optimal contract should appear as a reasonable compromise between the insurer's and the reinsurer's interests. Therefore, bargaining is required to reach a reinsurance contract, and it is rational to deal with this problem in a principal–agent framework. Boonen, Tan, and Zhuang (2016) show that, in contrast to indifference pricing, bargaining contracts allow the insurer and the reinsurer to share the benefits from insurance. Insurance and reinsurance contracts are frequently examined in terms of game theory (Boonen et al., 2016; Doherty & Smetters, 2005; Kihlstrom & Roth, 1982; Quiggin & Chambers, 2009). However, these studies, unfortunately, do not state how to price unit insurance risk.

Young and Zariphopoulou (2002) examine dynamic price of insurance risk by applying the principle of equivalent utility. However, they only consider prices that the buyer and the seller are willing to receive, and ignore how to reach an agreement on the price. Emms and Haberman (2005), Emms (2007), and Emms, Haberman, and Savoulli (2007) assume that the general insurance premium is a multiple of the market average premium, which is a stochastic process, and regard the multipliers as the relative insurance prices. Henriet, Klimenko, and Rochet (2016) adopt the relative safety loading in the variance principle to represent the dynamics of insurance prices. They solve optimal dynamic insurance prices by maximizing insurers' objective functions for given demand functions.

Inspired by these studies, we extend the relative safety loading in the expected value principle to represent the reinsurance price. In the traditional expected value principle and variance principle, the relative safety loadings of reinsurance are pre-specified constants. This leads to reinsurance premiums being certain per-unit

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https://doi.org/10.1016/j.ejor.2017.10.061

exposure per unit of time, irrespective of how much reinsurance is purchased. These premium principles fail to consider the bargaining between the insurer and the reinsurer, or the reinsurer's profit. In this study, the reinsurer maximizes his objective function over the relative safety loading of reinsurance in the expected value principle, and the loading changes with the reinsurance demand. In contrast to the above studies, our demand functions and price strategies are determined by bargaining.

Moreover, we introduce uncertainty (model ambiguity, model misspecification) into the classic Cramer–Lundberg (C–L) claims process. In practice, limited data or heterogeneous expert opinions make a decision maker difficult to specify an accurate probability for the stochastic risk (Balbás, Balbás, Balbás, & Heras, 2015; Ben-Tal, Bertsimas, & Brown, 2010; Borgonovo & Marinacci, 2015; Ellsberg, 1961). Insurance and reinsurance companies are asked to quote prices for hypothetical contracts in which the probabilities of losses are ambiguous (Cabantous, Hilton, Kunreuther, & Michel-Kerjan, 2011; Chen, Joslin, & Tran, 2012). Gilboa and Schmeidler (1989) point out that ambiguity-averse decision-makers consider alternative models that are difficult to distinguish statistically from the reference model. Branger and Larsen (2013) show that there are obvious differences between the uncertainty about the jump risk and that of the diffusion risk. The latter has been studied extensively. Here, we assume that the reinsurer faces ambiguity about the jump risk. Then, using a continuous-time framework allows us to describe the reinsurer's belief distortions by perturbations of the reference model. To model the reinsurer's preference with regard to ambiguity, we use the penalty-based multiple-priors utility proposed by Anderson, Hansen, and Sargent (2003) and Maenhout (2004), which allows us to separate the perception of ambiguity and attitude to ambiguity.

The goals of this study are to design robust proportional and excess-loss reinsurance contracts in a continuous-time principal–agent framework. We assume that the reinsurer copes with model uncertainty by designing a reinsurance contract that maximizes his utility in the worst-case scenario, subject to the insurer's incentive constraint. Since the contracts satisfy the incentive compatibility constraints, they are always executable. The insight derived from our results is that the reinsurer's aversion to model uncertainty generates an endogenous belief distortion, because he pessimistically believes that the true claim intensity is greater than the predicted value. Thus, the reinsurance price is always distorted upward relative to the standard price without model ambiguity, which reduces the demand for reinsurance.

Although we only consider a simple principal–agent relationship, and ignore the likely existence of adverse selection and moral hazard, we still derive several novel conclusions. First, in contrast to the representative agent model, in which the insurer and reinsurer are simply price takers and reinsurance demand decreases continuously, in the principal–agent framework, the reinsurer dynamically decreases the reinsurance price in order to increase the demand for reinsurance. Indeed, the insurer's purchase increases over time. Second, the price of excess-loss reinsurance is obviously higher than that of proportional reinsurance. In other words, in an actual application, the reinsurer should specify a higher relative safety loading for the excess-loss reinsurance contract than he would for the proportional reinsurance contact if he needs to use constant loadings. Despite this, the higher price cannot totally offset the tail risk of the excess-loss reinsurance contract faced by the reinsurer. Third, the risk-averse insurer does not always prefer excess-loss reinsurance. When his risk aversion is sufficiently small or the reinsurer's risk aversion is large, the excess-loss reinsurance contract is dominated by the proportional reinsurance contract. Finally, the reinsurer's risk aversion lowers the insurer's utility loss from choosing the proportional reinsurance contract, but his ambiguity aversion raises the utility loss. This reveals that the reinsurer's ambiguity aversion is no longer observationally equivalent to increasing his effective risk aversion.

The remainder of this paper is organized as follows. Section 2 presents the principal–agent framework and introduces the reinsurer's belief distortions. Sections 3 and 4 solve the robust proportional and excess-loss reinsurance contracts, respectively. Section 5 quantifies the effects of risk and ambiguity attitudes on the robust reinsurance contracts, and analyzes which reinsurance contract is favorable for the insurer and the reinsurer by means of numerical experiments. Lastly, Section 6 concludes the paper. Appendix A and Appendix B provide proofs of the main results.

2. Model setup

Consider a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, P)$, where $T>0$ represents the contract period. Filtration $(\mathcal{F}_t)_{t\in[0,T]}$ is generated by a compound Poisson process $\sum_{i=1}^{N(t)} Z_i$, and includes all available information until time t. Here, all processes are assumed to be progressively measurable with respect to $\mathcal{F}_t$, and expectations are satisfied the regularity conditions to ensure them well-defined.

In this section, we formulate the proportional reinsurance contract in a continuous-time agent–principal framework, in which a risk-averse insurer is the agent and a risk-ambiguity-averse reinsurer is the principal.

2.1. The agent's problem

We first state the insurer's problem. Suppose that the insurer's risk process $L(t)$ is modeled by the classic C–L model,

$$dL(t) = cdt - d\sum_{i=1}^{N(t)} Z_i, \tag{1}$$

where $c>0$ is the premium rate, and $\sum_{i=1}^{N(t)} Z_i$ is a compound Poisson process representing the cumulative claims up to time t. The claim times $\{N(t)\}_{t\in[0,\,T]}$ follow a homogeneous Poisson process with intensity $\lambda>0$. The claim sizes $\{Z_i, i\geq 1\}$ are independent and identically distributed positive random variables. Suppose that Z_i is independent of $N(t)$, and has distribution function $F(Z)$ and probability density function $f(Z)$. Denote the finite mean value $E[Z_i]=\mu$.

The insurer controls his claims risk by purchasing reinsurance, and is allowed to invest his surplus in a risk-free money account with constant interest rate r. Insurance and reinsurance premiums are computed by the expected value principle. However, in contrast to traditional studies, the relative safety loading of reinsurance in the expected value principle is not pre-specified, but is adjusted as demand and time vary. Since higher loadings imply more expensive insurance and reinsurance, we extend the relative safety loadings of insurance and reinsurance to represent their respective prices. The reinsurer offers the insurer a proportional reinsurance contract that specifies a reinsurance price $\eta=\{\eta(t): 0\leq t\leq T\}$ and a suggested risk retention proportion $a=\{a(t)\in(0,1): 0\leq t\leq T\}$. Then, for the price $\eta(t)$, the reinsurance demand is $1-a(t)$. In the presence of reinsurance, we can express the insurer's wealth dynamics as

$$\begin{aligned} dW(t) &= rW(t)dt + [(1+\theta)\lambda\mu - (1-a(t))(1+\eta(t))\lambda\mu] \\ &\quad dt - a(t)d\sum_{i=1}^{N(t)} Z_i \\ &= [rW(t) + (\theta-\eta(t))\lambda\mu + a(t)(1+\eta(t))\lambda\mu] \\ &\quad dt - a(t)d\sum_{i=1}^{N(t)} Z_i, \end{aligned} \tag{2}$$