ARTICLE IN PRESS
JID: EOR [m5G;November 13, 2017;9:12]

European Journal of Operational Research 000 (2017) 1–14

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Stochastics and Statistics

Bounds on risk-averse mixed-integer multi-stage stochastic programming problems with mean-CVaR

Ali İrfan Mahmutoğulları, Özlem Çavuş*, M. Selim Aktürk

Department of Industrial Engineering, Bilkent University, Ankara 06800, Turkey

ARTICLE INFO

Article history:
Received 2 July 2016
Accepted 17 October 2017
Available online xxx

Keywords:
Stochastic programming
Mixed-integer multi-stage stochastic programming
Dynamic measures of risk
CVaR
Bounding

ABSTRACT

Risk-averse mixed-integer multi-stage stochastic programming forms a class of extremely challenging problems since the problem size grows exponentially with the number of stages, the problem is non-convex due to integrality restrictions, and the objective function is nonlinear in general. We propose a scenario tree decomposition approach, namely group subproblem approach, to obtain bounds for such problems with an objective of dynamic mean conditional value-at-risk (mean-CVaR). Our approach does not require any special problem structure such as convexity and linearity, therefore it can be applied to a wide range of problems. We obtain lower bounds by using different convolution of mean-CVaR risk measures and different scenario partition strategies. The upper bounds are obtained through the use of optimal solutions of group subproblems. Using these lower and upper bounds, we propose a solution algorithm for risk-averse mixed-integer multi-stage stochastic problems with mean-CVaR risk measures. We test the performance of the proposed algorithm on a multi-stage stochastic lot sizing problem and compare different choices of lower bounds and partition strategies. Comparison of the proposed algorithm to a commercial solver revealed that, on the average, the proposed algorithm yields 1.13% stronger bounds. The commercial solver requires additional running time more than a factor of five, on the average, to reach the same optimality gap obtained by the proposed algorithm.

1. Introduction

In risk-averse stochastic optimization problems, risk measures are used to assess the risk involved in the decisions made. Due to the structural properties of risk measures, risk-averse models are more challenging than their risk-neutral counterparts. The multi-stage risk-averse stochastic models are even more complicated due to their dynamic nature and excessive amount of decision variables. Both the risk-neutral and risk-averse multi-stage stochastic problems are non-convex when some of the decision variables are required to be integer valued. Therefore, the solution methods suggested for convex multi-stage stochastic problems cannot be used to solve these problems.

In this study, we consider risk-averse mixed-integer multi-stage stochastic problems with an objective function of dynamic mean conditional value-at-risk (mean-CVaR). Both CVaR and mean-CVaR are coherent measures of risk that have been used in the literature extensively (see, Rockafellar & Uryasev, 2002). Coherent measures of risk and their axiomatic properties are introduced in the pioneering paper by Artzner, Delbaen, Eber, and Heath (1999). Later, the theory of coherent risk measures is extended by Ruszczynski and Shapiro (2006a, 2006b), and references therein.

In a multi-stage decision horizon, risk involved in a stream of random outcomes is considered. Therefore, dynamic coherent risk measures are introduced to quantify the risk in multi-stage models (see, Artzner, Delbaen, Eber, Heath, and Ku, 2007; Kovacevic and Pflug, 2009; Pflug and Römisch, 2007; Ruszczynski and Shapiro, 2006a; 2006b, and references therein).

For the multi-stage stochastic optimization problems with dynamic measures of risk, some exact solution techniques are suggested under the assumption that the decision variables are continuous. These techniques, such as stochastic dual dynamic programming (SDDP), which is first suggested by Pereira and Pinto (1991) for risk-neutral problems and then extended to risk-averse problems by Shapiro (2011), Shapiro, Tekaya, da Costa, and Soares (2013), Philpott, de Matos, and Finardi (2013), and Lagrangian relaxation of nonanticipativity constraints suggested by Collado, Papp, and Ruszczyński (2012) rely on the convex structure of the problem, therefore, they cannot be used to find an exact solution when some of the decision variables are integer valued. On the other hand, these methods can be used to obtain lower bounds on the optimal value of multi-stage stochastic integer problems.

* Corresponding author.
E-mail address: ozlem.cavus@bilkent.edu.tr (Ö. Çavuş).

https://doi.org/10.1016/j.ejor.2017.10.038

Bonnans, Cen, and Christel (2012) propose an extension of SDDP method for the risk-neutral problems with integer variables by relaxing the integrality requirements in the backward steps of the algorithm. Later, Bruno, Ahmed, Shapiro, and Street (2016) extend this approach to risk-averse integer problems. Zou, Ahmed, and Sun (2016) consider SDDP method to solve risk-neutral multi-stage mixed-integer problems with binary state variables. They prove that SDDP method provides an exact solution to the problem in finite number of iterations when the cuts satisfy some sufficient conditions. Similarly, Schultz (2003) uses Lagrangian relaxation of nonanticipativity constraints to obtain lower bounds within a branch-and-bound procedure for risk-neutral multi-stage problems with integer variables. However, these approaches rely on some restrictive assumptions. SDDP method requires stagewise independency of random process and the branch-and-bound procedure requires complete recourse assumptions. Therefore, they cannot be applicable to a wide range of problems.

A recent stream of research proposes an alternative way of obtaining bounds for mixed-integer multi-stage stochastic problems via a scenario tree decomposition. In that approach, the sample space is partitioned into subspaces called as groups, and the problem is solved for the scenarios in a group instead of the original sample space. These smaller problems are called as group subproblems. Sandıkçı, Kong, and Schaefer (2013) propose a group subproblem approach for risk-neutral mixed-integer two-stage stochastic problems. They show that the expected value of the optimal values of group subproblems gives a lower bound on the optimal value of the original problem. Later, this approach is extended to the risk-neutral multi-stage problems by Sandıkçı and Özaltın (2014), Zenarosa, Prokopyev, and Schaefer (2014), and Maggioni, Allevi, and Bertocchi (2016). Recently, Maggioni and Pflug (2016) apply group subproblem approach to risk-averse mixed-integer multi-stage stochastic problems where the objective is a concave utility function applied to the total cost over the planning horizon. Although, group subproblems include less number of scenarios than the original problem, the length of the decision horizon in group subproblems and the original problem is the same. Therefore, one may argue that scalability is a drawback of this approach when the decision horizon is too long.

In this study, we propose a scenario tree decomposition algorithm for risk-averse mixed-integer multi-stage stochastic problems with a dynamic objective function defined via mean-CVaR. The suggested algorithm is based on group subproblem approach and is used to find lower and upper bounds on the optimal value of the problem. We propose infinitely many valid lower bounds on mean-CVaR risk measure that can be used within the frame of the algorithm. We also investigate the effect of scenario partitioning strategies on the quality of the different lower bounds by considering different partitioning strategies based on the structure of the scenario tree and disparateness of scenario realizations.

As outlined earlier, our approach does not require any special structural property such as convexity and linearity of the feasible set. Moreover, it does not require complete recourse or stagewise independence assumptions, therefore, it can be applied to a wide range of problems. We conduct computational experiments on a multi-stage lot sizing problem by considering different choices of bounds and scenario tree partitions. The experiments reveal that the obtained bounds are tight and require reasonable CPU times. Our approach yields 1.13% stronger bounds than solving the problem with IBM ILOG CPLEX. On the other hand, CPLEX requires more than 5.45 times of CPU time to obtain the same optimality gaps of our approach.

The organization of the paper is as follows: In Section 2, we present problem definition and some preliminaries. Section 3 includes our main results on obtaining different lower bounds for mean-CVaR via a scenario grouping approach. We consider the application of these lower bounds to a risk-averse mixed-integer multi-stage stochastic problem with a dynamic objective function defined via mean-CVaR. We also suggest a method to obtain an upper bound. The computational study conducted on a multi-stage lot sizing problem and related discussions are presented in Section 4. Section 5 is devoted to concluding remarks and future research directions.

2. Risk-averse mixed-integer multi-stage stochastic problems with dynamic mean-CVaR objective

We consider a multi-stage discrete decision horizon where the decisions at stage $t \in \{1, \ldots, T\}$ are made based on the available information up to that stage. Let Ω be a finite sample space and $\{0, \emptyset\} = \mathscr{F}_1 \subset \mathscr{F}_2 \subset \cdots \subset \mathscr{F}_T = \mathscr{F}$ be a filtration, that is, an ordered set of sigma algebras on Ω, representing gradually increasing information through stages. We use ξ_t and x_t to denote the vector of problem parameters and decisions at stage $t \in \{1, \ldots, T\}$, respectively. For each $t \in \{1, 2, \ldots, T\}$, ξ_t and x_t are $\mathscr{F}_t-$measurable. At first stage, the vector of problem parameters ξ_1 and decisions x_1 are deterministic, since $\mathscr{F}_1 = \{0, \emptyset\}$. At stage $t \in \{2, \ldots, T\}$, some or all problem parameters are random.

An element ω of Ω is called as a scenario. A scenario $\omega \in \Omega$ corresponds to a realization of a sequence of random parameters $\xi_2(\omega), \ldots, \xi_T(\omega)$ in stages $2, \ldots, T$.

Our main interest is a risk-averse mixed-integer multi-stage stochastic problem with an objective of dynamic risk measure $\varrho_{1,T}(\cdot)$ over the horizon $1, \ldots, T$. The problem can be defined as:

$$\min_{x \in \mathcal{X}} \varrho_{1,T}(f_1(x_1), f_2(x_2, \xi_2), \ldots, f_T(x_T, \xi_T)), \tag{1}$$

where $\mathcal{X} = \mathcal{X}_1 \times \mathcal{X}_2(x_1, \xi_2) \times \cdots \times \mathcal{X}_T(x_{T-1}, \xi_T)$ is the abstract representation of possibly nonlinear feasibility set. Let $\mathbb{R}$ and $\mathbb{Z}$ denote the set of real numbers and integers, respectively. $\mathcal{X}_1 \subseteq \mathbb{R}^{n_1} \times \mathbb{Z}^{m_1}$ is a mixed-integer deterministic set and, for $t \in \{2, \ldots, T\}$, $\mathcal{X}_t : \mathbb{R}^{n_{t-1}} \times \mathbb{Z}^{m_{t-1}} \times \Omega \rightrightarrows \mathbb{R}^{n_t} \times \mathbb{Z}^{m_t}$ are $\mathscr{F}_t-$measurable mixed-integer point-to-set mappings. The cost in the first stage is deterministic and represented by a possibly nonlinear, real-valued function $f_1 : \mathbb{R}^{n_1} \times \mathbb{Z}^{m_1} \to \mathbb{R}$. The cost functions $f_t : \mathbb{R}^{n_t} \times \mathbb{Z}^{m_t} \times \Omega \to \mathbb{R}$, $t \in \{2, \ldots, T\}$ are $\mathscr{F}_t-$measurable, real-valued, and may be nonlinear.

Classical solution methods such as SDDP and Lagrangian relaxation of nonanticipativity constraints cannot be used to solve problem (1) due to integrality restrictions of some decision variables. Therefore, our focus is to obtain bounds on (1) where the objective function $\varrho_{1,T}(\cdot)$ is a dynamic risk measure defined via mean-CVaR.

Now, we present some necessary concepts and notation on coherent, conditional, and dynamic risk measures to exploit the structure of problem (1).

2.1. Coherent measures of risk

Let $\mathcal{Z} := \mathcal{L}_\infty(\Omega, \mathscr{F}, P)$ be the space of bounded and $\mathscr{F}$-measurable random variables with respect to sample space Ω and probability distribution P. Let $Z, W \in \mathcal{Z}$ represent uncertain outcomes for which lower realizations are preferable. Also, let Z_ω be the value that the random variable Z takes under scenario $\omega \in \Omega$. As defined in Artzner et al. (1999), a function $\rho : \mathcal{Z} \to \mathbb{R}$ is called a coherent measure of risk if it satisfies:

(A1) Convexity: $\rho(\alpha Z + (1-\alpha)W) \leq \alpha\rho(Z) + (1-\alpha)\rho(W)$ for all $Z, W \in \mathcal{Z}$ and $\alpha \in [0, 1]$,
(A2) Monotonicity: $Z \succeq W$ implies $\rho(Z) \geq \rho(W)$ for all $Z, W \in \mathcal{Z}$,
(A3) Translational Equivariance: $\rho(Z + t) = \rho(Z) + t$ for all $t \in \mathbb{R}$ and $Z \in \mathcal{Z}$,
(A4) Positive Homogeneity: $\rho(tZ) = t\rho(Z)$ for all $t > 0$ and $Z \in \mathcal{Z}$,

Please cite this article as: A.İ. Mahmutoğulları et al., Bounds on risk-averse mixed-integer multi-stage stochastic programming problems with mean-CVaR, European Journal of Operational Research (2017), https://doi.org/10.1016/j.ejor.2017.10.038