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Decision Support

An exact solution method for binary equilibrium problems with compensation and the power market uplift problem

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ABSTRACT

We propose a novel method to find Nash equilibria in games with binary decision variables by including compensation payments and incentive-compatibility constraints from non-cooperative game theory directly into an optimization framework in lieu of using first-order conditions of a linearization, or relaxation of integrality conditions. The reformulation offers a new approach to obtain and interpret dual variables to binary constraints using the benefit or loss from deviation rather than marginal relaxations. The method endogenizes the trade-off between overall (societal) efficiency and compensation payments necessary to align incentives of individual players. We provide existence results and conditions under which this problem can be solved as a mixed-binary linear program.

We apply the solution approach to a stylized nodal power-market equilibrium problem with binary on-off decisions. This illustrative example shows that our approach yields an exact solution to the binary Nash game with compensation. We compare different implementations of actual market rules within our model, in particular constraints ensuring non-negative profits (no-loss rule) and restrictions on the compensation payments to non-dispatched generators. We discuss the resulting equilibria in terms of overall welfare, efficiency, and allocational equity.

1. Introduction

There are many real-world settings where several players interact in a non-cooperative game with binary decisions, such as electricity markets (on-off decision for a power plant), transportation and facility location models (Caunhye, Nie, & Pokharel, 2012), engineering (Rao, 1996), as well as agriculture and land-use planning (Tóth, Haight, & Rogers, 2011). Modeling Nash equilibria between players which face both binary and continuous decisions is a challenging problem (Scarf, 1990). Economists and game theorists usually apply brute-force methods by exploring all possible combinations and check every solution for deviation incentives of each player. When market-clearing prices to support a pure-strategy Nash equilibrium in the Walrasian sense do not exist, economists suggest to use multi-part pricing (Hotelling, 1938) or deviate from marginal-cost pricing to a "second-best" market outcome, such that no player should lose money from participating (Baumol & Bradford, 1970). However, a canonical approach to find pure-strategy Nash equilibria in binary games does not exist.

In many large-scale practical applications, exploring the entire solution space is not realistically possible. A common approach in such cases is to linearize the binary decisions; the Nash equilibrium can then be computed by solving the system of first-order optimality conditions, a.k.a. equilibrium modeling using mixed complementarity problems or variational inequalities, if certain assumptions on convexity of the linearized problem hold. Recent work seeks a trade-off between relaxation of the complementarity (slackness) conditions or the integrality of discrete constraints to obtain stationary points that are presumed to be equilibria of the original problem (Fuller & Celebi, 2016; Gabriel, Conejo, Ruiz, & Siddiqui, 2013; Gabriel, Siddiqui, Conejo, & Ruiz, 2012).

In this work, we focus on applications where a relaxation of optimality conditions or continuous relaxation of the binary decision variable ("linearization") is either not practical or yields incorrect results. Instead, we derive first-order optimality conditions of the continuous variables for *both states of each binary variable* and include those in an overall equilibrium problem simultaneously. Our method then selects the state of the binary

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variable and corresponding continuous variable which provides the best response for each individual player.

Due to the nature of a binary game, there are many instances where no set of strategies and no price vector exists that supports a Nash equilibrium in pure strategies; i.e., there is no outcome where the pay-offs to each stakeholder are such that no player has a profitable deviation. This is due to the non-convexity introduced by the binary decision variables and indivisibilities (O'Neill, Sotkiewicz, Hobbs, Rothkopf, & Stewart Jr, 2005). We introduce the notion of a "quasi-equilibrium" to describe situations where no equilibrium exists, but where a market operator or regulator can assign compensation payments in order to obtain an incentive-compatible outcome. These payments align the incentives of individual players with the objectives of the overall system, such as cost minimization or welfare maximization. A regulator may also choose to intervene when an equilibrium exists but its outcome is inferior to the solution that a benevolent planner might achieve. That is, the market operator may seek to minimize the deviation from the system optimum (i.e., all decisions by one planner) caused by the non-cooperative game among a number of decision makers, each seeking to optimize competing objectives. Our solution approach allows to endogenously consider the trade-off between regulatory intervention to improve market efficiency, and the distortions caused by these interventions.

Electricity markets are the real-world application of binary games which have received the most attention in the mathematical optimization literature (Bjørndal & Jörnsten, 2008; Hu & Ralph, 2007; Liu & Hobbs, 2013; Liu & Ferris, 2013; O'Neill, Krall, Hedman, & Oren, 2013; O'Neill et al., 2005; Philpott, Ferris, & Oren, 2013; Philpott & Schultz, 2006; Wogrin, Hobbs, Ralph, Centeno, & Barquín, 2013). A challenging problem arises from the on-off decision of power plants, which usually incur substantial start-up or shut-down costs and, if operational, face minimum-generation constraints. Because power markets are usually based on marginal-cost, short-term pricing, the commitment costs (i.e., start-up costs) are not necessarily covered by resulting market prices.

As a consequence, many electricity systems have rules that generators must be "made whole" or have to be "in the money"; i.e., they receive "uplift payments" to make sure that they do not lose money from participating in the market. This is commonly referred to as a "no-loss rule". However, this may not be required from a game-theoretic point of view, and thereby lead to higher-than-necessary compensation payments. At the same time, there might exist regulations that only power plants that are actually generating electricity can receive compensation – the rationale being that it may create perverse incentives for market participants to be paid to not do something. We will discuss and illustrate in a numerical example how such market rules can actually overly restrict operational efficiency and thereby reduce welfare.

The outline of this paper is as follows: in the next section, we summarize current approaches to solve binary Nash games and place our contribution in the context of methods applied to solve such problems in the power sector. In Section 3, we propose an exact solution method to solve binary equilibrium problems. The obtained multi-objective program explicitly incorporates the trade-off between overall efficiency and compensation payments in cases where no equilibrium exists. Section 4 applies our method to a power market example from the literature to illustrate its advantages and flexibility to incorporate distinct market rules regarding uplift payments. Section 5 concludes with a discussion on methods, other possible applications, and future work.[1]

2. Current approaches to solve binary games

In this section, we motivate our method by describing how current solution methods for binary games obtain equilibria, and we identify where our formulation can improve this process. While there exist brute-force methods (Audet, Belhaïza, & Hansen, 2006; Avis, Rosenberg, Savani, & Von Stengel, 2010; Von Stengel, 2002) that solve for an equilibrium considering all possible combinations of the binary variables and check ex-post for deviation incentives, we want to concentrate on mathematical programming techniques for obtaining equilibria. For large-scale applications such as those considered in this work, computational efficiency proves a hurdle in these brute-force methods. Solving a large number of equilibrium problems is not very elegant and suffers from a curse of dimensionality, because the number of equilibrium problems to be solved is 2^k, where k is the number of binary variables. Therefore, mathematicians and Operations Researchers are constantly looking for ways to apply advances in Variational Inequalities and Integer Programming to develop faster methods to solve such problems.

2.1. Optimization and equilibrium modeling

Game theory and equilibrium problems have been an integral part of the history of mathematical programming. First-order optimality (Karush–Kuhn–Tucker, KKT) conditions, derived from each individual player's optimization problem, can be solved simultaneously by stacking them to form an equilibrium problem. Interpretations from dual variables to constraints in a game theory analysis provide essential information in equilibrium problems and are often interpreted as prices or marginal benefits for individual players (Facchinei & Pang, 2003; Ferris & Pang, 1997; Murphy, Sherali, & Soyster, 1982).

However, this relationship between optimality conditions and equilibrium problems fails once a game includes binary decision variables. The reason is that optimality conditions cannot be directly derived for binary optimization problems. Thus, applied researchers aim to solve such optimization problems in other ways. A method based on a trade-off between relaxing the integrality and the complementarity constraints is developed by Gabriel et al. (2013). While relaxing integrality has been employed as a way to solve integer programs, relaxing complementarity – essentially the optimality conditions – was the novel idea of their contribution.

A similar problem is tackled by Fuller and Celebi (2016); they propose a *minimum disequilibrium* model, defining disequilibrium as the difference between the pay-off in the socially optimal outcome and the individually optimal decision, summed over all players. That is, they seek to minimize the aggregated opportunity costs for all market participants from following the instructions of a social planner. The authors relate the MD model both to the results obtained by a social planner and to the model proposed by Gabriel et al. (2013).

One alternative recent method to tackle binary equilibrium problems focuses on solving integral Nash–Cournot games (Todd, 2014) and provides an efficient algorithm to obtain equilibria. This method works very well for a specific integer game with no constraints, but the algorithm is not applicable to the broad class of binary-constrained games considered in this paper.

2.2. Dual variables in binary programs

As mentioned above, dual variables in constrained convex optimization contain useful information both for computational

[1] The Appendix provides computational results for a numerical test case using a larger data set than the stylized example in Section 4. The GAMS codes for the stylized example, the numerical test case, as well as an additional example for a resource market application with multiple binary investment decisions in production and pipeline capacity for several player are available for download at https://github.com/danielhuppmann/binary_equilibrium under a *Creative Commons Attribution 4.0 International License.*