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Innovative Applications of O.R.

Two-dimensional cutting stock problem with sequence dependent setup times

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ABSTRACT

Motivated by a firm in the technical textile industry, we study a two-dimensional cutting stock problem with sequence dependent setup times and permissible tolerances. We provide a sequential heuristic with feedback loop based on the approach of Gilmore and Gomory and formulate the sequencing problem as a mixed integer program. We derive a lower bound algorithm and demonstrate the near-optimal performance of our heuristic. Finally, we use real data to test our heuristic and illustrate its applicability to a problem of realistic size.

1. Introduction

Firms across many industries face cutting problems, for instance in the paper, steel, textile, and glass industries (Delorme, Iori, & Martello, 2016; Yanasse & Lamosa, 2007). These problems are typically formalized as instances of the Cutting Stock Problem (CSP) with the objective of identifying a set of patterns leading to minimal trim loss when cutting products out of larger items (Cui, Zhong, & Yao, 2015). Gilmore and Gomory (1961) were among the first to propose a solution procedure for the one-dimensional CSP where products only differ in width. Their approach dynamically generates new patterns that are good candidates for the optimal solution (Kallrath, Rebennack, Kallrath, & Kusche, 2014). Since then, many extensions have been suggested (cf. Wäscher, Haußner, and Schumann (2007) and Lübbecke and Desrosiers (2005) for excellent taxonomies and reviews). However, besides few exceptions (e.g., Reinertsen & Vossen, 2010; Rinaldi & Franz, 2007; Yanasse & Lamosa, 2007; Yuen & Richardson, 1995), these algorithms only generate optimal patterns, but do not provide production plans indicating optimal sequences (Reinertsen & Vossen, 2010). Moreover, for many one-dimensional problems neither the question where to produce products within a pattern nor where to optimally locate knives was relevant, so patterns only indicated which products ought to be produced together. In two- or more-dimensional applications of the cutting stock problem, the issue is relevant and has been studied under the objective of waste minimization.

While early work pursued the objective of waste minimization, the focus has increasingly shifted to more general objectives that better reflect the needs of production planners. Song, Chu, Nie, and Bennell (2006) emphasize in their approach that production costs comprise not only material costs but also production time. Indeed, setup times are a key aspect of a series of papers acknowledging that increased productivity may be more valuable than marginal reductions in waste, particularly when freed-up capacity can be used for additional orders. Common modeling approaches either aim to limit the number of different cutting patters (Umetani, Yagiura, & Ibaraki, 2003) or penalize the use of different patterns by including a fixed per-pattern cost in the objective function (Cui et al., 2015; Haessler, 1975; Mobasher & Ekici, 2013). Cui et al. (2015) propose heuristics for the one-dimensional CSP with per-pattern setup times and demonstrate strong performance of their heuristics across diverse data sets. However, sequence dependent setup times have not yet been reflected in such approaches.

As an important aspect of production planning in the CSP context, sequencing has been studied under two similar objectives. Dyson and Gregory (1974) as well as Yuen and Richardson (1995) consider the objective of minimizing the queue of partially cut orders, as each partially completed order may cause discontinuities in production. Yanasse (1997) as well as Yanasse and Lamosa (2007) seek to minimize the number of open stacks. Their objective is thus to complete orders of products with the same dimension soon after their production start to keep the number

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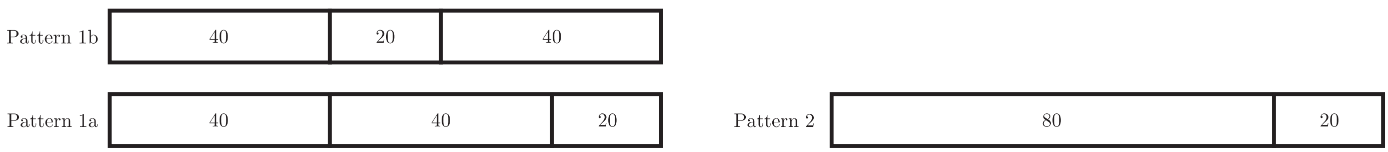

Fig. 1. Patterns and pattern classes where Patterns 1a and 1b have the same length.

of different compartments in the production plan small. However, neither of them considers sequence dependent setup times.

1.1. Problem description

The problem addressed in our work relates to this literature and is motivated by a firm in the technical textile industry with a tight capacity constraint, which suggested as objective the minimization of total production time. Deterministic demand needs to be satisfied by producing fabric tapes characterized through width and length. As products are anisotropic, they cannot be rotated. To understand the key elements of production time, consider an important process in the technical textile industry: weaving. In this process, fabric is formed by rectangularly lateral interlacing with longitudinal threads (Choogin, Bandara, & Chepelyuk, 2013). The time required for this weaving process is the first element of total production time.

While the length of fabrics (longitudinal direction) may vary to match product dimensions, the width of the fabric (lateral direction) is determined by the width of the machine. To allow for thinner products, fabrics can be cut into tapes in the longitudinal direction using knives. To insert a knife, several longitudinal threads have to be removed as they would otherwise end up moving around loosely in the machine. Similarly, when a knife is removed, several threads have to be inserted. Both processes are quite cumbersome and, in our context, take about 20 minutes each. Only one knife can be moved at a time. This can lead to eight hours of downtime in the worst case, whereas only 20 minutes are needed when only one knife needs to be inserted or removed. Time for knife insertions or removals is the second component of total production time.

Once the fabric is cut into tapes, these are spooled on an axis which is then removed from the machine. Unless the fabric length matches the length of a demanded product, these tapes are rewound onto another axis of appropriate width and cut in lateral direction to match the desired product length. This rewinding is done an a different machine for which capacity is no issue. Removing axes from the weaving machine, which is called unmounting, is time consuming and forms the third and last additive term of total production time. Since the objective is the minimization of production time rather than waste minimization, overproduction is allowed as it may reduce the number of knife movement. The overproduced products are considered as waste.

In short, the problem is a two-dimensional CSP with guillotine cuts (Gilmore & Gomory, 1965) that comprises sequence dependent setup times. The latter requires finding a sequence of patterns as well as positions of knives at each step of this sequence so as to minimize the required number of knife movements. To account for the knife positions in our terminology, we call the specification of product quantities with a specific, lateral arrangement of tapes and their associated knives a *pattern*. A *pattern class* is a set of patterns where each pattern leads to the same tapes but may have different tape arrangements. Since lateral arrangements of tapes have not been of importance to former studies, the term 'pattern' has here simply been used to express the number of products to be produced on an axis (cf., Gilmore and Gomory, 1963, who use 'column' and 'pattern' interchangeably).

Consider Fig. 1. As Patterns 1a and 1b lead to the exact same tapes, both belong to the same pattern class. Previous literature on CSP and sequencing treats both patterns structurally as identical. For instance, in terms of Yanasse (1997) as well as Yanasse and Lamosa (2007) both patterns require the same stacks be opened. Or in terms of Dyson and Gregory (1974) as well as Yuen and Richardson (1995) they would be related to the same open orders or discontinuities. Importantly, in our context, if Pattern 2 follows Pattern 1a only one knife needs to be removed. If Pattern 2 follows Pattern 1b two knives need to be removed and one inserted, thus tripling the required setup time. The distinction between patterns and pattern classes is thus important and distinguishes the problem studied in this paper from problems considered in previous studies. A summary of the key terminology is provided in Table 1.

There are two further aspects of the problem that can lead to a substantial increase in the number of possible combinations and thus the complexity of the problem. First, industry norms allow for tolerances that depend on the width of tapes. Second, the sum of all tape widths within a pattern may not add up to the width of an axis leading to flexibility of arranging the products.

1.2. Structure of the paper

In Section 2, we model a version of the two-dimensional CSP with sequence dependent setup times and tolerances to minimize total production time, comprising time for actual weaving production, knife setups, and axis mounting. We formulate a mixed integer linear program that yields a production plan which specifies the products to be produced jointly on axes as well as the arrangement of products on each axis. In Section 3 we propose a sequential heuristic that identifies suitable pattern classes for use as inputs for a sequencing model, which again is formulated as a mixed integer linear program. Results of this program are, in turn, fed back to identify even better pattern classes in the first phase. In order to assess the performance of this procedure, we also provide an algorithm to derive a lower bound. In Section 4 we numerically compare the performance of the full model with the heuristic and illustrate the performance of our heuristic based on real data from a firm in the technical textile industry from 2013 and 2014. In Section 5, we conclude and provide suggestions for further research.

2. Cutting stock model with sequence dependent setup times and tolerances

Consider n_P products, indexed by $p \in \mathcal{P} = \{1, \ldots, n_P\}$, with known demands, $\left(d_p\right)_{p\in\mathcal{P}}$, that must be satisfied. Products differ in terms of norm width, $\left(w_p\right)_{p\in\mathcal{P}}$, and length, $\left(l_p\right)_{p\in\mathcal{P}}$. Without loss of generality, we group products with the same dimensions together, that is, if $w_p = w_q$ and $l_p = l_q$ then $p = q$. A notational overview is given in Table A.1 in Appendix A.

We assume that the number of theoretically available axes, n_A, is sufficient to facilitate any production plan: $n_A = \sum_{p\in\mathcal{P}} d_p$. Let all

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