European Journal of Operational Research 254 (2016) 398–409

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

Robust efficiency measures for linear knapsack problem variants

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ARTICLE INFO

Article history:
Received 29 June 2015
Accepted 13 April 2016
Available online 21 April 2016

Keywords:
Combinatorial optimization
Heuristic
Genetic Algorithm
Knapsack problem

ABSTRACT

Numerous combinatorial optimization applications, such as the mobile retailer product mix problem, utilize the multidemand, multidimensional knapsack problem variant in which there are multiple knapsack constraints requiring a weighted summation of the variables to be less than or equal to a nonnegative value and multiple demand constraints requiring a weighted summation of the variables to be greater than or equal to a nonnegative value. The purpose of this paper is to demonstrate that core variables and efficiency measures, concepts used in the most efficient solvers to-date for binary knapsack problems and some of its variants, can be extended to the multidemand, multidimensional knapsack problem variant. Specifically, new efficiency measure calculations are provided and their properties are mathematically proven and experimentally demonstrated. The contribution of such measures to knapsack problem research is that these measures are applicable to all knapsack problem variants with a single linear objective function and linear constraints of any quantity. The applicability of these new measures is demonstrated through the development of three heuristic procedures: a Fixed-Core heuristic, a Genetic Algorithm, and a Kernel Search heuristic. The results from these tests are compared with the results from a commercial solver and an existing heuristic. The findings from these tests demonstrate that the Fixed-Core and Kernel Search heuristics developed for this paper are the most efficient solvers to-date for hard multidemand, multidimensional knapsack problems.

1. Introduction

Recent advances in solving Knapsack Problems (KPs) have utilized the concepts of core variables and efficiency measures. The concept of core variables for KPs was first introduced by Balas and Zemel (1980) who observed that only a small percentage of the decision variable values in a KP change between the optimal binary solution and the optimal linear relaxation solution. The variables whose values differ between these two solutions were then defined as the ‘core variables’. Further experimentation indicated that the quantity of core variables increases at a less than linear rate with respect to the number of decision variables (Pisinger, 1999). Hence, identifying these variables can greatly reduce the computational effort when solving the binary problem, especially for large problem instances.

The obvious challenge with identifying these core variables is that it requires knowledge of the solution to the binary KP which defeats the purpose of exactly identifying this set. However, research has identified closed form ‘efficiency measures’ which use the problem-specific coefficients to rank the variables such that those variables most likely to be in the core are clustered together. A majority of modern KP techniques utilize these measures to create extremely efficient solution procedures, both exact and heuristic, for solving the binary KP and some of its variants.

To date, efficiency measures have been developed for the basic KP as well as for the multidimensional KP (MKPs) variant which has multiple knapsack constraints. These measures are typically a ratio of the objective coefficient for the variable over the weighted sum of the constraint coefficients (Puchinger, Raidl, & Pferschy, 2010). One of the preferred weighting techniques in these measures is to use the optimal dual variable values from the linear relaxation solution as they serve as a dependent measure of each constraint’s importance with respect to the objective function.

The disadvantage of these measures is that they have strict assumptions with regards to problem structure. With respect to this research, the first and most important assumption in current efficiency measures is that there are no demand constraints in the problem. Within KP terminology, a demand constraint is any constraint which has a weighted sum of decision variables which must be greater than or equal to a given threshold. The most universal KP variant which contains these constraints is the multidemand MKP (MDMKP) which has multiple knapsack and

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http://dx.doi.org/10.1016/j.ejor.2016.04.025
0377-2217/

multiple demand constraints. The second assumption for current efficiency measures is that the objective and constraint coefficients are non-negative. Such an assumption is typically valid for MKPs, but it may no longer be valid for MDMKPs as including items with a negative objective coefficient may be necessary to satisfy demand constraints and including items with a negative knapsack constraint coefficient may not be automatic depending on the values of the variable's other coefficients. Hence, new measures are needed if more complex KP variants are to be solved using recent KP solution techniques.

Solving the MDMKP is motivated by its applicability to other KP problems as well as its applications to documented problems in the literature. The principal application of interest for this research is in determining the ideal product mix for a mobile retailer which is constrained by a traditional KP constraint modeling the space of the retailer as well as by a constraint for requiring the product mix to meet or exceed a given revenue threshold (Wishon & Villalobos, 2016). Additional applications include selecting an optimal project portfolio (Beaujon, Marin, & McDonald, 2001), locating and routing for obnoxious facilities (Cappanera, Gallo, & Maffioli, 2004), and the sea cargo mix problem (Ang, Cao, & Ye, 2007).

The principal goal of this article is to demonstrate that closed-form efficiency measures exist for the MDMKP. The benefit of such measures is that they are robust efficiency measures for all KP variants with a single linear objective function and linear constraints with any type of equality or inequality since these formulations can be transformed into a MDMKP. The developed robust efficiency measures will then be employed in three solution techniques to demonstrate the efficiency measures' utility in solving MDMKPs. These solution techniques were selected as they demonstrate that the new measures can be used to extend existing KP solution techniques to solve MDMKPs.

The remainder of the article is as follows. Section 2 summarizes the most relevant MDMKP solution methodologies as well as the most recent solution methods for all KP variants that utilize efficiency measures. Section 3 introduces the new efficiency measures and mathematically proves their properties. Section 4 experimentally demonstrates that the robust efficiency measures provide the same benefit for MDMKPs as the traditional measures do for simpler KPs while Section 5 introduces three heuristic solution procedures which utilize the new measures. Finally, Section 6 concludes with a summary of all completed work along with a discussion of algorithmic shortcomings and future research goals.

2. Background

The literature in this section covers two subsections of modern KP research. First, a summary is provided on the use of core variables and efficiency measures in KPs with specific emphasis on the use of efficiency measures for KP variants. This is followed by a review of the current literature on solving MDMKPs. Those techniques ranking variables with pseudo-efficiency measures will be given special attention.

The concept of core variables for traditional, binary KPs was first introduced by Balas and Zemel (1980) who observed that dynamic programming solutions for KPs relied on a complete sorting and branch and bound algorithm for all of the decision variables even though only a small interval of the sorted variables differed between the linear relaxation solution and the optimal binary solution. Balas and Zemel used this observation to create a partial sorting methodology and heuristic which was able to quickly develop good solutions for the binary KP. This sorting was completed by using the ratio of the objective coefficient over the constraint coefficient for each variable, a technique first employed by Dantzig (1957) to solve the linear KP. These ratios have since been employed as the standard efficiency measures for traditional, binary KPs.

Martello and Toth (1988) then used this partial sorting to create an efficient, exact solution procedure which first assumed an approximate core prior to reducing the problem and solving the remainder through a depth-first branch and bound procedure. The next advancement was from Pisinger (1995b) whose algorithm completed a sorting-as-needed, depth-first, branch and bound that prioritized branching based on the variables most likely to be in the core as determined from their efficiency measures. Pisinger (1997) then updated this algorithm to use a breadth-first approach which proved to be superior. At the same time, Martello and Toth (1997) developed their own algorithm for solving hard KPs using strong bounding rules. These last two works were then combined by Martello, Pisinger, and Toth (1999) to create the fastest exact solution method for binary KPs to date.

Other recent advances have focused on expanding the core variable and efficiency measure concepts to KP variants. The greatest contribution of such research is in the development of efficiency measures for problems with multiple knapsack constraints. These types of measures were first introduced by Dobson (1982) who used a measure which was the ratio of the objective coefficient over the sum of the constraint coefficients. These measures have since been updated to feature a weighted sum of the constraint coefficients, typically weighted by the optimal dual variables, as demonstrated by Angelelli, Mansini, and Grazia Speranza (2010), Puchinger et al. (2010), and Della Croce and Grosso (2012). In addition, either the efficiency measure for MKPs or the measure for standard KPs has been used to solve other KP variants including an equality KP (Volgenant & Marsman, 1998), bounded KP (Pisinger, 2000), unbounded KP (Martello & Toth, 1990), multiple-choice KP (Pisinger, 1995a), multiple-choice MKP (Ghasemi & Razzazi, 2011), and multi-objective KP (Gomes da Silva, Clímaco, & Rui Figueira, 2008; Lust & Teghem, 2012; Mavrotas, Rui Figueira, & Florios, 2009). For those interested in more information, concise reviews exist for solving KPs or their variants using core approaches, either exactly (Dudziński & Walukiewicz, 1987; Martello, Pisinger, & Toth, 2000) or heuristically (Wilbaut, Hanafi, & Salhi, 2008).

While techniques for solving traditional KPs is substantial, research into solving MDMKPs is limited in comparison. The first focused research into a solution method for the MDMKP is from Cappanera and Trubian (2005) who developed a tabu-search heuristic which searches the near-infeasible solution space for feasible solutions which are then used as seeds for local searches within the feasible region. A tabu-search procedure was later developed by Arntzen, Hvattum, and Løkketangen (2006) with greater focus on exploring the infeasible solution space compared with Cappanera and Trubian. Following this research, Hvattum and Løkketangen (2007) developed a scatter search method which found feasible solutions by performing various mathematical combinations of previously identified feasible solutions. Later, Hvattum, Arntzen, Løkketangen, and Glover (2010) developed an alternating control tree procedure which uses the linear relaxation solution to create subproblems of the initial MDMKP. The research by Hvattum et al. is unique as it is the sole solution method which can determine the optimal binary solution so long as an optimal solution procedure is applied to the subproblem. Another solution procedure is from Balachandar and Kannan (2011) whose dominance-based heuristic ranks variables based on the value of constraint coefficients such that efficient additions/removals of variables are made from the final solution set. While such an approach does rank the variables, it does not utilize all problem information such as the objective coefficients or the relative importance of each constraint. Hence, none of the aforementioned research applies efficiency measures or core variables to improve their solution procedures.