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A new time-independent reliability importance measure

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ABSTRACT

Modern digital systems pose new challenges to reliability analysts. Systems may exhibit a non-coherent behavior and time becomes an important element of the analysis due to aging effects. Measuring the importance of system components in a computationally efficient way becomes essential in system design. Herein, we propose a new importance measure for time-independent reliability analysis. The importance measure is based on the change in mean time to failure caused by the failure (success) of a component. It possesses some attractive properties: *i)* it is defined for both coherent and non-coherent systems; *ii)* it has an intuitive probabilistic and also geometric interpretation; *iii)* it is simple to evaluate. It turns out that the proposed importance measure leads naturally to a test of time consistency. We illustrate the properties with examples of coherent and non-coherent systems. A comparison with the ranking of other time-dependent and time-independent reliability importance measures is also offered. The realistic application to the reliability analysis of an H.264 video decoder concludes the work.

1. Introduction

The constantly shrinking sizes of modern semiconductor devices enable the design and production of ever smaller, more efficient, and also more economical computing systems. However, these small device structures are increasingly susceptible to environmental effects like cosmic rays, manufacturing tolerances, and in particular aging effects such as negative/positive bias temperature instability or electromigration. Consequently, reliability is nowadays not only a prime design objective in safety-critical fields of application, but plays a crucial role in the design of all kinds of computing systems, and particularly in the design of embedded systems (see Borkar, 2005). However, while for safety-critical systems regulatory requirements may already justify the additional costs needed to achieve the desired level of safety, for systems typical of the embedded domain we need to look for highly cost-efficient techniques to compensate for the outlined increasing unreliability of system components.

Given these trade-offs, reliability importance measures have traditionally helped designers in redundancy allocation. The complexity of today's systems makes manual design often prohibitive, and calls for automatic approaches to explore the design spaces of possible system implementations/configurations. Recently, design space exploration techniques have shown that also automatic optimization approaches can benefit from importance measures (see Aliee, Glaß, Khosravi, & Teich, 2014; Khosravi, Reimann, Glaß, & Teich, 2014). Given the complexity of the systems involved, importance measure analysis needs to be carried out in a computationally efficient way. Aging effects are increasingly affecting components performance introducing the need of modeling system reliability in a time-dependent fashion. In order to apply most of the existing reliability importance measures to this scenario, the designer has to chose a certain point in time (e.g., the mission time of the system) at which the importance of components is evaluated. This constraint imposes not only the need for careful manual interaction—which we aim to avoid—but, even more, the burden of choosing the right point in time. In fact, the importance of components may significantly change over time, if we employ time-dependent importance measures. As a remedy, we investigate and address time-independent reliability importance measures in this work. However, most time-independent reliability importance measures have been defined for coherent systems.

Now, because embedded systems typically share constrained resources between different applications, they may show non-coherent behavior. For example, the activation (or repair) of redundant software tasks on a processor may delay the computation causing a certain deadline to be missed and then inducing a system failure. Consequently, a time-independent reliability importance measure is required that can be applied to coherent as well as non-coherent systems and whose computation is numerically

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efficient. Since we seek for an importance measure of seamless applicability, we introduce an intuitive concept, that is, we measure the importance of a component as the magnitude of the difference between the conditional and unconditional system Mean Time To Failures (MTTFs) given that the component has failed. Both values are readily available to the system designer via classic reliability analysis approaches, such as binary decision diagrams or event-fault trees. Both values, also, are well defined for both coherent and non-coherent systems. We then show that under the condition of time consistency (a special case of stochastic dominance which we are to discuss later on), the importance measure coincides with the Wasserstein–Kantorovich distance between the unconditional and conditional reliability functions. The fact that this is the area enclosed between the two distribution functions provides the importance measure also with an intuitive geometric interpretation. Finally, because aging effects may be accelerated by common disruptive agents such as environmental heat, components may no longer be considered independent but may exhibit dependent failures. In this respect, the proposed importance measure does not require that component failures are independent.

Aside from discussing the new importance measure, we shall present a brief excursion into the concept of time consistency, defined as a second-order stochastic dominance condition. We show that time consistency can be possessed by both a coherent or non-coherent system. Such concept is therefore independent of the structure of the system.

While the previous discussion concerns component failure, we also consider the twin importance measure based on component success. The importance measure quantifies the impact of making the component perfectly reliable on the system MTTF and possesses similar properties to the importance measure for failures.

We illustrate the new importance measure(s) and the concept of time consistency with alternative examples and conclude the work with a realistic digital system application (an H.264 video decoder) from the domain of automatic embedded system design.

The reminder of the work is organized as follows. Section 2 discusses related work on reliability importance measures. The novel time-independent reliability importance measure is introduced in Section 3. The concept of time consistency and the geometric interpretation of the new importance measure are discussed in Section 4. Section 5 illustrates several examples of coherent and non-coherent systems. Section 6 presents a realistic application. Section 7 Presents a comparison of time dependent versus time independent importance analysis. Section 8 offers conclusions and future research perspectives.

2. A review of time-dependent and time-independent importance measures

In this section, we offer an overview of the literature relevant to our work. Because our focus is on importance measures, they will be investigated in greater detail in this section. But, relevant items to our work are also time-dependent reliability analysis and system modeling. The literature on these subjects is vast and we can only offer a cursory review. As for time-dependent reliability analysis, we recall the works of Barlow and Proschan (1975b), Rackwitz (2001), Boudali and Bechta Dugan (2006) and Prescott, Remenyte-Prescott, Reed, Andrews, and Downes (2009). As for system modeling, we recall the works of Epstein and Rauzy (2005), Wang and Trivedi (2005) and, recently, Dutuit and Rauzy (2015).

We now come to importance measures, to which the longer part of this review section is dedicated. Reliability importance measures are analytical tools developed to quantitatively appraise the contribution of a component to system reliability. Since the seminal work of Birnbaum (1969), a variety of reliability importance measures have been introduced, to cope with alternative sensitivity questions that emerge in alternative applications. The works of Aven and Norkland (2010), Kuo and Zhu (2012b), Vasseur and Llory (1999), and the monograph by Kuo and Zhu (2012a) offer broad overviews.

Of interest is the state of a given system at time t. The system is made of n elements (components or basic events), whose states determine the end-state of the system. In the remainder, we shall use the term basic event or component exchangeably. This end state is called top event. Thus, we let:

$$\varphi_i = \begin{cases} 1 & \text{component } i \text{ has failed} \\ 0 & \text{component } i \text{ is working correctly} \end{cases} \tag{1}$$

denote the state variable of a generic basic event, and $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_N)$ is then the basic event state vector. The generic configuration φ can result in the system to be working or failed. We denote the top-event indicator variable as Ψ, with $\Psi = 1$ denoting system failure.

The relationship that binds the state vector φ to the system state variable Ψ is called structure function. Technically, $\Psi(\varphi)$ can be regarded as a Boolean function or as a state function (Crama & Hammer, 2011). We will regard it as a Boolean function and use the notation of Boolean logics hereafter. A state φ is an implicant if it implies $\Psi(\varphi) = 1$. It is a prime implicant if it does not contain any other implicant. In the logic of failure/success, prime implicants are minimal cut/path sets, respectively. In reliability studies, of interest for a decision-maker is the probability that the system performs (or does not perform) its mission over the assigned time horizon. For simplicity, assume that the system is not repairable. Such probability is called the reliability of the system at time t, and is denoted as:

$$R(t) = \Pr(\Psi = 0; t) \tag{2}$$

The unreliability of the system is denoted here as

$$U(t) = \Pr(\Psi = 1; t) = 1 - \Pr(\Psi = 0; t) = 1 - R(t). \tag{3}$$

A central intuition for the definition of several reliability importance measures is the notion of criticality. A component is said to be critical if the system is in such a state that:

$$\Psi(1_i, \varphi_{\sim i}) - \Psi(0_i, \varphi_{\sim i}) = 1, \tag{4}$$

that is, a switch from working to failed of component i provokes the occurrence of the top event. Here, $\boldsymbol{\varphi}_{\sim i}$ is the basic event state vector of the system without component i. Then, the Birnbaum importance measure is defined as:

$$I^B = \Pr[\Psi(1_i, \varphi_{\sim i}) - \Psi(0_i, \varphi_{\sim i})] = \mathbb{E}[\Psi(1_i, \varphi_{\sim i}) - \Psi(0_i, \varphi_{\sim i})]. \tag{5}$$

Birnbaum (1969) shows that, if: *a)* the system is coherent, and *b)* we assume that basic event occurrences are probabilistically independent, then

$$I^B = \frac{\partial U}{\partial F_i} \tag{6}$$

where $F_i = \Pr(\varphi_i = 1)$ is the probability that basic event i realizes. By Eq. (6), the Birnbaum importance of a basic event is time dependent, because the system unreliability U is time dependent. The definition in Eq. (6) holds for coherent systems. Andrews and Beeson (2003) propose an extension of this definition for non-coherent systems.

Barlow and Proschan (1975a) introduce a modification of the Birnbaum importance which leads to the well-known time-independent Barlow–Proschan importance measure:

$$I_i^{BP} = \int_0^\infty \Pr[\Psi(1_i, \varphi_{\sim i}) - \Psi(0_i, \varphi_{\sim i})] dF_i(t), \tag{7}$$

where $F_i(t)$ is the cumulative distribution function of the failure time of component i. A notable probabilistic interpretation of the